OCTM 2009

1. Simplify: $2(0 + (0 + 9))$

2. What name do we give to a line that intersects a circle in exactly two points?

3. Find the <u>sum</u> of all the positive integral factors of 2009.

4. Find the slope of the line $200x - 9y = 2009$.

5. This stem-and-leaf plot (also called a stemplot) represents the number of hot dogs consumed by contestants during an eating contest. Let x be the mean and y be the median. Find $x - y$.

2	0 0 9
3	9
4	1 6 7 8
5	1 7

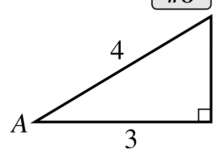
Key: 3 | 5 = 35

6. Express in simplest non-factored form: $4a^3b + 2ab^3 - 4a^3b + 2(ab^3)^0$

7. Let $\mathbb{Z}$ represent the integers, $\mathbb{Q}$ the rational numbers, $\mathbb{R}$ the real numbers, and $\mathbb{R}^*$ the irrational numbers. Circle "T" (true) or "F" (false) for each of the following statements:

   a) $\mathbb{Q} \subset \mathbb{R}^*$ b) $\mathbb{Q} \subset \mathbb{R}$ c) $\mathbb{Q} \subset \mathbb{Z}$

#8



8. See figure. Give the <u>exact</u> value of $\sin A$.

9. Let x be the GCF of 8 and 10, and let y be the LCM of 8 and 10. Find xy.

10. The point at $(2, 0)$ is rotated $150°$ counterclockwise about the origin. Find the exact coordinates of its image.

11. Suppose x and y vary inversely. If $x = 2009$ when $y = 10$, find y when $x = m$.

12. Write the <u>capital letter</u> of the answer that completes this statement: A triangle is divided into two triangles of equal area by ____.

   A) an altitude B) an angle bisector C) a median
   D) a midsegment E) a perpendicular bisector of a side

13. Math teacher Carol Botzner found the value of the absolute value expression $|3 - 8|$. Math teacher Gene Meister found the value of the modulus $|3 - 8i|$. Math teacher/musician Susan Cantey found the value of the determinant $\begin{vmatrix} 8 & 3 \\ 3 & 8 \end{vmatrix}$. Write the <u>product</u> of these three values.

14. Find the number three-elevenths of the way from -9002 to 2009.

15. Solve for all real values of x: $4x^3 = -25x$.

16. Four mathematics teachers are conducting a probability experiment. A bowl contains 5 red marbles and 3 blue marbles. Carol Oberholtzer randomly pulled out a marble, saw that it was blue, and replaced it. James Fanger randomly pulled out a marble, saw that it was blue, and replaced it. Then Mike Bruns randomly pulled out a marble, saw that it was blue, and replaced it. Now Fred Dillon randomly pulls out a marble. What is the probability that Mr. Dillon's randomly-drawn marble is blue?

17. Write the <u>capital letter</u> of the equation that represents the conic section called a <u>hyperbola</u>.

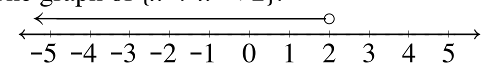
   A) $4y^2 + 4x^2 = 9$ B) $y = -3x^2 + 9$ C) $9xy = 16$
   D) $\dfrac{x^2}{25} + \dfrac{y^2}{16} = 1$ E) $\dfrac{x^2}{25} - \dfrac{y}{16} = 1$

18. Suppose x is $\frac{200}{9}$ of y and y is $\frac{9}{200}$ of z. If $x = 1800$, find z.

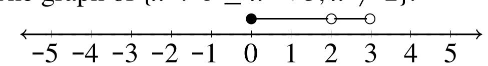
19. The numbers m and n are consecutive positive integers, and $m < \sqrt{100} \le n$. What is the value of mn?

20. Here are two examples of graphs on number lines:

   The graph of $\{x : x < 2\}$:



   The graph of $\{x : 0 \le x < 3, x \ne 2\}$:



   Using the number line provided on the answer sheet, show the set $\{x : |x - 1| < (x - 1)^2\}$.

21. I. M. Rich invested \$10,000 at simple interest for one year, part at 6% and the rest at 5%. If the total amount of interest earned after one year was \$520.09, how much money was invested at 5%?

22. Circle "T" (true) or "F" (false) for each of the following statements:

   a) 100 feet < 100 meters b) 100 centimeters < 100 inches
   c) 100 pounds < the weight of 100 kilograms on Earth

23. Suppose a and b are real numbers, and $\dfrac{a + bi}{1 + i} = \dfrac{7}{7 + i}$. Find the value of the sum of $a + b$.

24. Consider the following pairs of quantities:

	Column I	Column II	
i.	$f(g(4))$	$g(f(4))$	where $f(x) = 3x - 2$ and $g(x) = 2x - 3$
ii.	$\dfrac{x-1}{x}, x \ne 0$	$\dfrac{x}{x-1}, x \ne 1$	

   For each part, determine the relationship between the two quantities and write for your answer:

   A if the quantity in Column I is greater C if the quantities are equal
   B if the quantity in Column II is greater D if none of A, B, or C is true

25. The polynomial function $f(x)$ satisfies $f(0) = 36$. The equation $f(x) = 0$ has a double root of 2 and a root of $3i$. Write this polynomial function in the form $f(x) = ax^4 + bx^3 + cx^2 + dx + e$, where a, b, c, d, e are real numbers.

26. Solve for all θ such that $0° \le \theta° < 360°$: $4\cos^2\theta - 1 = 0$.

27. The <u>geometric</u> mean of two numbers is 10, and the <u>arithmetic</u> mean of the same two numbers is 26. Find the <u>larger</u> of these two numbers.

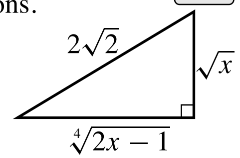
28. A rectangle has an area of 2009 square inches. Find the smallest possible perimeter of that rectangle.

29. Give an example of two positive integers x and y, $2 < x < y < 12$, that are relatively prime, yet neither one of them is prime. Write your answer as an ordered pair (x, y).

30. Three mathematics teachers were working problems at the OCTM annual conference.
   - Vainard Spiess drew a regular hexagon with circumradius 100 cm.
   - James Velo drew a regular quadrilateral with an apothem of 100 cm.
   - Brett Miller drew a regular triangle with an altitude of $100\sqrt{3}$ cm.

   Find the sum of the lengths of the perimeters of these three regular polygons.

#31



31. See figure. Given a right triangle with side measures as shown, solve for all values of x.

32. $\log_5 25 = x$ and $\log_{25} 5 = y$. Find xy.

33. A point is 9 inches from a circle, and the length of the tangent drawn to the circle from this point is 24 inches. Find the length of the diameter of the circle.

34. Express 2009 in base 9.

35. Three non-zero numbers a, b, c form an arithmetic progression. Increasing a by 1 or increasing c by 2 results in a geometric progression. Find the value of b.

36. Huey, Louie, and Dewey are brothers. Louie is 3 years older than Dewey and 14 years older than Huey. If Huey's age were tripled, the result would be 5 more than the sum of the present ages of Louie and Dewey. How old is Dewey?

37. Find all positive integral values of n for which the expression $\dfrac{n^3-12}{n-4}$ has an integral value.

38. This woman was born in 1820 and died in 1910. Her father believed women—especially his daughters—should get an education, so she and her sister learned Italian, Latin, Greek, history, and mathematics. She was an innovator in the collection, tabulation, interpretation, and graphical display of descriptive statistics, although she is better known in a field different from mathematics. During the Crimean War in 1854, she found that soldiers wounded in battle were ten times more likely to die from illnesses contracted in the hospital (such as typhus, typhoid, cholera and dysentery) than from their wounds. She developed the "polar-area diagram" to dramatize the needless deaths caused by unsanitary conditions and the need for reform. She used the information from these statistics to bring about great reform in the sanitary conditions of hospitals. Name this woman.

39. A given trapezoid has perpendicular diagonals. If one of these diagonals has a length of 15, and the altitude of the trapezoid has a length of 12, find the area of the trapezoid.

40. Superman flew d miles at r miles per hour but arrived 2 hours late. How fast should he have flown to get there on time?

THE OHIO COUNCIL OF TEACHERS OF MATHEMATICS

Thirty-sixth Annual Contest

February 28, 2009

You may write on this test. Please keep this test when you finish.

During the test, each student is permitted to have one or more handheld calculators, **including** the TI-89, TI-92, TI-*n*spire, Voyage 200, and HP95. Calculators with cordless transmission capabilities must be taped over. **No cell phones, PDAs, laptops, or other electronic devices allowed in the test room. If you have one, give it to your coach or test monitor before the test begins, or your score may be voided.**

On the front of the answer sheet, print your first name, middle initial, and last name. Please check the information on the back of the answer sheet, and correct if necessary.

Instructions:

- Place each answer on its proper blank on the answer sheet.
- There may be one or more questions which are impossible. For the purpose of this test, write "impossible" or "no solution" or $\varnothing$ or { }. **NO CREDIT** given for $\{\varnothing\}$.
- There may be one or more questions with multiple answers. In such cases, all answers are required unless specified otherwise.
- Write multiple solutions as (e.g.) "{2, 3}" **OR** "$x = 2$ or $x = 3$" **OR** "2, 3".
  NO CREDIT given for ordered pair form "(2,3)".
  NO CREDIT given for "$x = 2$ **AND** $x = 3$."
- In problems 1–20, **EXACT ANSWERS IN SIMPLEST FORM** are necessary.
  For example: write "$1+\sqrt{2}$" (not 2.414...); write "$\frac{\pi}{4}$" (not 0.785398...);
  write "$x = 5$ or $x = 1$" (not 3 ± 2); write "1" (not x^0);
  write "$\frac{4}{9}$" or "$0.\overline{4}$" (not $\frac{16}{36}$, nor $(\frac{2}{3})^2$, nor 0.4444).
- After problem 20, unless otherwise specified, the answer may be written in **exact decimal, radical or fractional form,** or decimal form **rounded off to four places to the right of the decimal point**.
- The questions are not arranged according to difficulty. (There are some easy problems after number 30. Check it out!)
- Testing time: **60 MINUTES**

Grading:

- Each correct answer counts one point. No partial credit will be given.
- There is no penalty for guessing.