

OCTM 2008

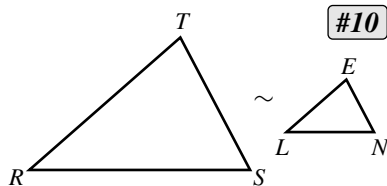
- Simplify: $200 + 8 + 2^0 - 0^8$.
- Find the arithmetic mean of 2008 and 8002.
- If the ratio of girls to boys in an algebra class is 3 : 2, and there are 30 students in that class, how many boys are in that class?
- A certain figure is defined to be the set of all points in a plane that are at a given distance from a given point in that plane. What is the geometric name given to this figure?
- The sum of 8 consecutive even natural numbers is 2008. Find the largest of those 8 natural numbers.

- Which one of these five is not equal to the other four expressions?

A) $(2x^2 - 3x - 3) + (2x^2 - 2x - 3)$	D) $(6x^2 + 4x - 4) - (2x^2 + 9x + 2)$
B) $3x^2 - 6x - 7 + x^2 + x + 1$	E) $(5x^2 - 6x - 7) - (x^2 + x - 1)$
C) $(4x + 3)(x - 2)$	

- List all the Fibonacci numbers less than 100 that are also prime numbers.
- If an aircraft carrier uses 1 gallon of fuel to travel 4 inches, how many gallons of fuel would be needed to travel between ports that are 1268 miles apart?
- The 3rd term of an arithmetic series is 8, and the common difference is $\frac{1}{2}$. Find the 9th term.

- Vanna said, " $\triangle RST \sim \triangle LNE$." If $RS = 12$, $TS = 8$, $RT = 10$, and $LN = 4$, find the perimeter of $\triangle LNE$.

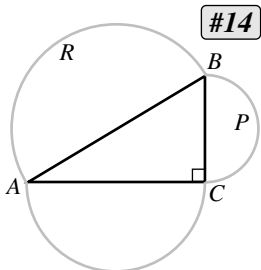


- Give an example of the equation of a line whose graph intersects quadrants I, II, and III, but not IV. Express in the form $y = mx + b$.

- A rectangle has a perimeter of 2008 cm. Find the greatest possible area for the rectangle. Be sure to include the correct units.

- If 2 is a solution (root) of $x^3 + kx + 10 = 0$, then find the value of k .

- (See figure.) Each side of right $\triangle ABC$ is the diameter of a semicircle, as shown. If the area of semicircle R is 50π and the area of semicircle P is 18π , find the perimeter of $\triangle ABC$.



- If $i = \sqrt{-1}$, find both solutions to $z^2 = -i$. Write your solutions in the form $a + bi$.

- Bonnie won the 3-way presidential election with 2008 votes, which was $\frac{4}{3}$ times as many as the total number of votes of her opponents Jim and Steve. If Jim received 50% as many votes as Steve, how many votes did Steve receive?

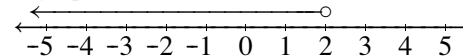
- Let " $\#$ " be defined as $a \# b = 4a - 3b + ab$. If $a \# b = 32$, and a and b are positive integers, find three possible values for a .

- Let $x = 0.1234567891011121314 \dots 997998999$, where the digits are obtained by writing the integers 1 through 999 in order. Find the 2008th digit to the right of the decimal point.

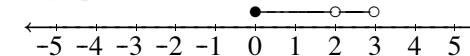
- Express the sum $0.35 + 0.3\bar{1}$ as a fraction in simplest form.

- Here are two examples of graphs on number lines:

The graph of $\{x : x < 2\}$:



The graph of $\{x : 0 \leq x < 3, x \neq 2\}$:



Using the number line provided on the answer sheet, show the graph of the set $\{x : x^3 < x^2\}$.

- Last year a bicycle cost \$200 and a cycling helmet cost \$50. This year the cost of the bicycle increased by 5%, and the cost of the helmet increased by 10%. Find the percent increase in the combined cost of the bicycle and the helmet from last year to this year.

- List the capital letter of all the following statements that is/are true. (All answers must be correct to receive credit.)

- | | |
|---|--------------------------------------|
| A) The maximum value of the sine function is 1. | D) $\sin(-x^\circ) = -\sin(x^\circ)$ |
| B) The value of the secant function is never 0. | E) The cosine function is odd. |
| C) The period of the tangent function is 2π . | |

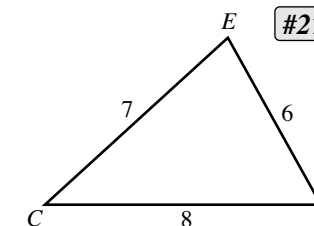
- What is the smallest positive integral value of k for which $35x^2 + kx + 18 = 0$ will have two distinct real number solutions?

- For how many integers n between 1 and 100, inclusive, does $x^2 + x - 2n$ factor into the product of two linear factors with integer coefficients?

- Find matrix products (a) AB and (b) BA , given matrices $A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 3 \\ -2 & 3 & 2 \end{bmatrix}$. (Both answers must be correct to receive credit.)

- Six marbles—numbered 1, 2, 3, 4, 5, 6—are placed in a box and drawn one at a time (without replacing the marbles). What is the probability that they are drawn in either numerical order or reverse numerical order?

- (See figure.) Find the area of the circle that is circumscribed about $\triangle CDE$.



- Find the number of distinct points common to the graphs of $x^2 + y^2 = 4$ and $y^2 = 4$.

29. Three machines A, B, and C, working together, can do a job in h hours. When working alone, A needs an additional 6 hours to do the job; B needs an additional one hour; and C needs an additional h hours. Find the value of h .
30. A group of mathematics teachers was grading a packet of OCTM Contest test papers, which had scores of 33, 25, 26, 17, 29, 38, 24, 25, and 36. Donna Preston found the mean, Stan Perry found the median, and Peggy Wielenberg found the mode. And then Brian McCombs found the mean of these newly-found three numbers. What was the mean found by Brian McCombs?

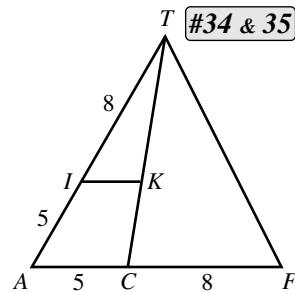
31. List the capital letters of the two equations that represent perpendicular lines.
- A) $3y - 2x = 12$ B) $-2x - 3y = 10$ C) $3y + 2x = 12$ D) $2y + 3x = 10$

32. In the expansion of $\left(m - \frac{1}{\sqrt{m}}\right)^7$, find the coefficient of $m^{-1/2}$.

33. Solve for all real values of x : $4 \log_8 x = \log_8 2x$.

34. (See figure.) Find the ratio of the area of $\triangle FAT$ to the area of $\triangle CAT$. Express in simplest form, $\frac{a}{b}$, where a and b are integers.

35. (See figure.) If \overline{KI} is parallel to \overline{CA} , find the ratio of the area of $\triangle KIT$ to the area of $\triangle CAT$. Express in simplest form, $\frac{c}{d}$, where c and d are integers.



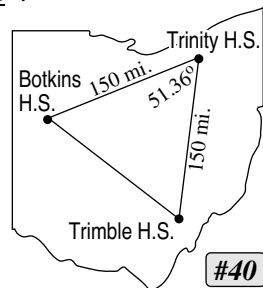
36. A *lattice point* is a point in the plane with integer coordinates. How many lattice points are on the line segment whose endpoints are (6, 13) and (31, 153)? (Include both endpoints of the segment in your count.)

37. Which one of these four points of a triangle does not necessarily lie on the triangle's Euler line?
 centroid circumcenter incenter orthocenter

38. In the history of this annual OCTM State Mathematics Contest, the only year in which the “contest number” was a factor of the “contest year” was its inaugural year—1974. That is, 1 is a factor of 1974, but 2 is not a factor of 1975, 3 is not a factor of 1976, 4 is not a factor of 1977, . . . , and 35 is not a factor of 2008. Assuming the contest continues annually, what is the next year in which the “contest number” will be a factor of the “contest year”?

39. For every x , $f(x^2 + 1) = x^4 + 5x^2 + 3$. Find $f(2x - 1)$.

40. At the OCTM Annual Conference, mathematics teachers Nancy Barile, Kevin Dael, and Mark Roggenkamp were looking at a map of Ohio. “Look at that,” said Nancy. “Our schools form an isosceles triangle.” Using the figure at the right, determine the area of the triangle formed, to the nearest square mile.



THE OHIO COUNCIL OF TEACHERS OF MATHEMATICS

Thirty-fifth Annual Contest February 23, 2008

You may write on this test. Please keep this test when you finish.

During the test, each student is permitted to have one or more handheld calculators, **including** the TI-89, TI-92, TI-*nspire*, Voyage 200, and HP95. Calculators with cordless transmission capabilities must be taped over. **No cell phones, PDAs, laptops, or other electronic devices allowed in the test room. If you have one, give it to your coach or test monitor before the test begins, or your score may be voided.**

On the front of the answer sheet, print your first name, middle initial, and last name. Please check the information on the back of the answer sheet, and correct if necessary.

Instructions:

- Place each answer on its proper blank on the answer sheet.
- There may be one or more questions which are impossible. For the purpose of this test, write “impossible” or “no solution” or \emptyset or $\{ \}$. **NO CREDIT** given for $\{ \emptyset \}$.
- There may be one or more questions with multiple answers. In such cases, all answers are required unless specified otherwise.
- Write multiple solutions as (e.g.) “{2, 3}” **OR** “ $x = 2$ or $x = 3$ ” **OR** “2, 3”. **NO CREDIT** given for ordered pair form “(2,3)”. **NO CREDIT** given for “ $x = 2$ **AND** $x = 3$.”
- In problems 1–20, **EXACT ANSWERS IN SIMPLEST FORM** are necessary. For example: write “ $1 + \sqrt{2}$ ” (not 2.414...); write “ $\frac{\pi}{4}$ ” (not 0.785398...); write “ $x = 5$ or $x = 1$ ” (not 3 ± 2); write “1” (not x^0); write “ $\frac{4}{9}$ ” or “0. $\overline{4}$ ” (not $\frac{16}{36}$, nor $(\frac{2}{3})^2$, nor 0.4444).
- After problem 20, unless otherwise specified, the answer may be written in **exact decimal, radical or fractional form**, or decimal form **rounded off to four places after the decimal point**.
- The questions are not arranged according to difficulty. (There are some easy problems after number 30. Check it out!)
- Testing time: **60 MINUTES**

Grading:

- Each correct answer counts one point. No partial credit will be given.
- There is no penalty for guessing.