

OCTM 2007

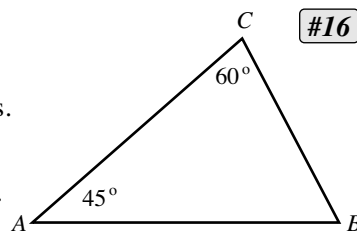
- Simplify: $2007 - 2 - 0 - 0 \cdot 7$.
- True or false? (All parts must be correct to receive credit.)
 - 2007 is a perfect square.
 - 2007 is a prime number.
 - 2007 is a rational number.
- What name do we give to a segment that connects any two consecutive vertices of a polygon?
- How many yards, feet, and inches are in (4 yds, 1 ft, 4 in) minus (1 yd, 2 ft, 7 in)?
- Which one of the following is not a valid postulate or theorem to prove that triangles are congruent? SSS, AAA, SAS, AAS, ASA
- Stubby Paycheck earned a gross income of \$500. 25% was deducted for taxes, and then \$100 was deducted for insurance. What percent of the original gross amount did Stubby have as net income?
- Using six of the digits 0–9 (no digit more than once), find the largest possible 6-digit odd number with a “9” in the ten’s place.
- Which of the following is one-half of 64^{4004} ?
 - 32^{4004}
 - 64^{2002}
 - 2^{24023}
 - 64^{4003}
 - 4^{12011}
- If $200x + 7 = 2007$, find the value of $7x - 200$.

- Find the 2007th digit after the decimal point in the decimal representation of $\frac{200}{7}$.
- In a bag are 6 sticks, with lengths of 3 cm, 4 cm, 5 cm, 6 cm, 7 cm, and 8 cm. If three of these sticks are picked at random, what is the probability that these three sticks could form the three sides of a triangle? Express as a fraction in simplest form.
- A polynomial function, $f(x) = ax^3 + bx^2 + cx + d$, has -3, 0, and 4 as zeros. Find b , c , and d when $a = 1$.
- Given $\cos(\theta) = \frac{3}{4}$, with the terminal side of θ in the 4th quadrant. Write the exact value of $\csc(\theta)$.

- How many lines of symmetry does a regular n -gon have?
 - n
 - $n - 1$
 - $n + 1$
 - $2n - 3$
 - It depends whether n is odd or even

- Mathematicus performed the following calculation: MDCLXVI \div XCVIII. Write the answer in Roman numerals.

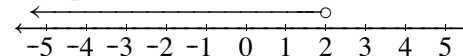
- The sum of the tangents of the 3 angles of this triangle is $3 + 2\sqrt{3}$. Find the product of the tangents of these 3 angles. Express in exact form.



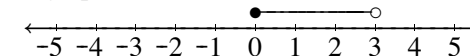
#16

- In base 3, write the number that is 8_{ten} more than 10101_{three} .
- Find the exact area of a triangle whose vertices are at (0, 3), (1, 4), and (2, 2).
- After five tests (on which possible grades were whole numbers ranging from 0 to 100), Matt Matticks has an average of exactly 88. Find the lowest score that Matt could possibly have received on any one of these tests.
- Here are two examples of graphs on number lines:

The graph of $\{x : x < 2\}$:

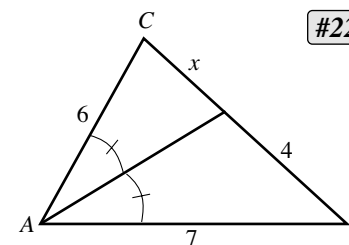


The graph of $\{x : 0 \leq x < 3\}$:



Using the number line provided on the answer sheet, show the graph of the union of the set $\{x : x^2 \leq 4\}$ with the set $\{x : x^2 > 1\}$.

- In 2003, I made x dollars. In 2004, I got a 10% raise. In 2005, I got a 9% raise. In 2006, I got an 8% raise. My 2006 salary represented what effective percent raise of my 2003 salary? Express your answer to the nearest tenth of one percent.
- (See figure.) Find the value of x .
- The 3rd term of a geometric series is 8, and the common ratio is $\frac{1}{2}$. Find the 7th term.

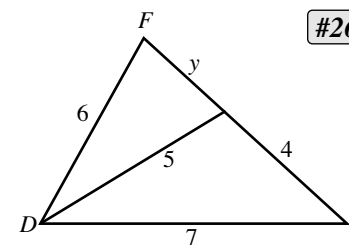


#22

- Which two of these statements require identical reasons (property and operation)?
 - $(a + b)c = ac + bc$
 - $(ab)c = (ba)c$
 - $a(bc) = (ab)c$
 - $(ab) + c = c + (ab)$
 - $(a + b) + c = a + (b + c)$
 - $(a + b) + c = (b + a) + c$

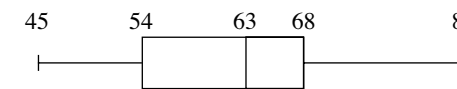
- In Mathburg, Ohio, $\frac{3}{4}$ of the adult females are married (to men) and $\frac{2}{3}$ of the adult males are married (to women). What fraction of the adult population in Mathburg is married? Express your answer as a fraction $\frac{a}{b}$.

- (See figure.) Find the value of y .
- How many permutations of the letters of the word “cassette” are there?
- Evaluate: $\log_3 27 - \log_4 4 + \log_{27} 3 - \log_{25} 1$.



#26

- In this box-and-whiskers plot (boxplot), find the value of the mean.
 - 63
 - 61
 - 62.4
 - 63.5
 - Cannot be determined
- If y varies directly as x and inversely as z , and $y = 60$ when $z = 10$ and $x = 40$, find y when $z = 30$ and $x = 45$.



31. The rules of Sudoku follow: Enter digits from 1 to 9 into the blank spaces so that every row, every column, and every 3×3 box contain every digit from 1 to 9 inclusive. At the right is the top one-third of a regular 9×9 Sudoku puzzle. Determine the product xy .

4	x	6	9	3		2		1
	3	1		7	2			6
9	y	7	1	5			4	8

32. The side of an equilateral triangle is 12. A circle is inscribed in the triangle, and a square is inscribed in the circle. Find the area of the square.
33. At the OCTM Conference, mathematics teacher Debbie Patonai bought 3 T-shirts at \$11 each, Janet Cummings bought 3 T-shirts at \$12 each, and Steve Lifer bought 4 T-shirts at \$13 each. What was the average cost per T-shirt?
34. Given $i = \sqrt{-1}$, solve for n : $(-2i^2)(2007) + \sum_{k=1}^2 (207k) - (2\sqrt{6})^2(27) + n = 7!$
35. Mathematics teacher Tom Lanning can grade a set of tests in 11 hours, Dolores Williams can grade a set of like tests in 12 hours, and Mary Farina can grade a set of like tests in 13 hours. How long will it take these three teachers working together to grade a set of like tests? Express your answer to the nearest minute.
36. Find the distance between the parallel lines $y = 0.5x + 2$ and $y = 0.5x + 4$.
37. A cube has the same total surface area as a rectangular solid whose dimensions are 16 inches long, 12 inches wide, and 9 inches high. Find the length of an edge of the cube. Be sure to include the correct units.
38. In 1614, John Napier discovered logarithms which made it possible to perform multiplication and division by adding and subtracting. Edmund Gunter used Napier's idea to draw a number line in which the positions of numbers were proportional to their logarithms. In about 1630 William Oughtred invented a mechanical analog computer, which was a precursor to the modern calculator. This instrument consists of at least 2 finely divided scales, most often a fixed outer pair and a movable inner one, with a sliding window called a cursor. This tool allowed the user to multiply, divide, find squares and square roots, logs and anti-logs. One of its nicknames became the "slipstick." Name this early mechanical calculator.

39. Given the ellipse $\frac{(x-3)^2}{4} + \frac{(y-4)^2}{25} = 1$, find the distance from an endpoint of the major axis to an endpoint of the minor axis.

40. Contest Director Charlene (Charlie) flew from Cincinnati to Cleveland to confer with former Contest Director Richard. Flying with a constant tailwind, she flew 280 miles in 2 hours. Flying against the wind, it took 2.5 hours to make the return trip. Find the plane's average air speed in miles per hour.



THE OHIO COUNCIL OF TEACHERS OF MATHEMATICS

Thirty-fourth Annual Contest
February 24, 2007

You may write on this test. Please keep this test when you finish.

During the test, each student is permitted to have one or more handheld calculators, **including** the TI-89, TI-92, Voyage 200, and HP95. Calculators with cordless transmission capabilities must be taped over. **No cell phones, PDAs, laptops, or other electronic devices allowed in the test room.**

On the front of the answer sheet, print your first name, middle initial, and last name. Please check the information on the back of the answer sheet, and correct if necessary.

Instructions:

- Place each answer on its proper blank on the answer sheet.
- There may be one or more questions which are impossible. For the purpose of this test, write "impossible" or "no solution" or \emptyset or $\{ \}$. **NO CREDIT** given for $\{ \emptyset \}$.
- There may be one or more questions with multiple answers. In such cases, all answers are required unless specified otherwise.
- Write multiple solutions as (e.g.) " $\{2, 3\}$ " **OR** " $x = 2$ or $x = 3$ " **OR** " $2, 3$ ". **NO CREDIT** given for ordered pair form " $(2,3)$ ". **NO CREDIT** given for " $x = 2$ **AND** $x = 3$."
- In problems 1–20, **EXACT ANSWERS IN SIMPLEST FORM** are necessary.
For example: write " $1 + \sqrt{2}$ " (not 2.414...); write " $\frac{\pi}{4}$ " (not 0.785398...);
write " $x = 5$ or $x = 1$ " (not 3 ± 2); write " 1 " (not x^0);
write " $\frac{4}{9}$ " or " $0.\overline{4}$ " (not $\frac{16}{36}$, nor $(\frac{2}{3})^2$, nor 0.4444).
- After problem 20, unless otherwise specified, the answer may be written in **exact decimal, radical or fractional form**, or decimal form **rounded off to four places after the decimal point**.
- The questions are not arranged according to difficulty. (There are some easy problems after number 30. Check it out!)
- Testing time: **60 MINUTES**

Grading:

- Each correct answer counts one point. No partial credit will be given.
- There is no penalty for guessing.