OCTM Contest Answer Sheet 2001 (back)

SOLUTIONS TO SELECTED PROBLEMS 2001

Typed Problems: Credit to Ashland University Professors Dr. Chris Swanson, Dr. Gordon Swain, Dr. Darren Wick
Typed by Dr. Chris Swanson

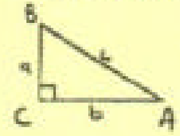
17) 16 Since $x+y=7$ and $(x+y)(x-y)=x^2-y^2=21$, $x-y=3$. Thus, $2x=(x+y)+(x-y)=7+3=10 \Rightarrow x=5$ and $y=2$. Hence, $2x+3y=2\cdot5+3\cdot2=16$.

23) $k=4n-5$ $(8^{4n})(32x)=4^{8n} \Rightarrow 2^{12n}\cdot2^5x=2^{16n} \Rightarrow 2^{12n+5}x=2^{16n} \Rightarrow x=\frac{2^{16n}}{2^{12n+5}}$

Thus, $x=2^{16n-(12n+5)}=2^{4n-5}$.

25) $\frac{3}{5}=.6$ $x+y=6, xy=10$. Then $\frac{1}{x}+\frac{1}{y}=\frac{y+x}{xy}=\frac{6}{10}=\frac{3}{5}$. OR $x=3+i$; $y=3-i$ Find $\frac{1}{3+i}+\frac{1}{3-i}$

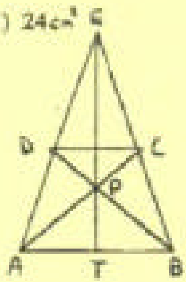
26) 2



$\sin^2 A+\sin^2 B+\sin^2 C=$

$\left(\frac{a}{c}\right)^2+\left(\frac{b}{c}\right)^2+(\sin 90°)^2=\frac{a^2+b^2}{c^2}+1=\frac{c^2}{c^2}+1=2$

31) $24cm^2$



Suppose E is the remaining vertex of the original isosceles triangle. Note the area of ΔDCE is the area of ΔABE minus the area of trapezoid $ABCD$. Thus, the area of ΔDCE is 18cm². ΔDCE is similar to ΔABE and has one fourth the area. Thus, the sides of ΔDCE are $\sqrt{\frac{1}{4}}=\frac{1}{2}$ as long as the sides of ΔABE. In particular, $ED=\frac{1}{2}EA$ and $EC=\frac{1}{2}EB$. Thus, $\overline{AC}$ and $\overline{BD}$ are medians of ΔABE. Two medians of a triangle meet at a point that is $\frac{1}{3}$ the distance along the remaining median from the remaining side to the remaining vertex. Thus, $PT=\frac{1}{3}ET$. Since ΔABE is isosceles, $\overline{ET}\perp\overline{AB}$. Thus,

$$\text{area of } \Delta ABP=\frac{1}{2}AB\cdot PT=\frac{1}{2}AB\cdot\frac{1}{3}ET=\frac{1}{3}\left(\frac{1}{2}AB\cdot ET\right)$$
$$=\frac{1}{3}\cdot\text{area of } \Delta ABE=\frac{72}{3}=24$$

32) $y=-\frac{4}{3}x+2$ The center of this hyperbola is (3, −2). One asymptote will pass through the center and the point $(3-\sqrt{9},-2-\sqrt{16})=(0,-6)$ and the other asymptote will pass through the center and the point $(3-\sqrt{9},-2+\sqrt{16})=(0,2)$. Clearly the second asymptote passes closer to the origin and its y-intercept is 2. The slope of this line is $\frac{-2-2}{3-0}=-\frac{4}{3}$. Thus, the equation of this asymptote is $y=-\frac{4}{3}x+2$.

33) $\frac{9}{2},\frac{27}{4}$ Let $a<b$ be the two numbers. Then since $3,a,b$ are in geometric progression,

$\frac{a}{3}=\frac{b}{a}\Rightarrow\frac{a^2}{3}=b$. Since $a,b,9$ are in arithmetic progression,
$b-a=9-b\Rightarrow 2b=9+a$. Thus,

$2\frac{a^2}{3}=9+a\Rightarrow 2a^2-3a-27=0\Rightarrow(2a-9)(a+3)=0\Rightarrow a=\frac{9}{2}$ (since $a>0$).

Then $b=\frac{(9/2)^2}{3}=\frac{81}{12}=\frac{27}{4}$.

34) −4 $x=\frac{[illegible]}{[illegible]}=\frac{88}{-22}=-4$

37) $4\sqrt{2}$ First note that $\angle DAB$ is a right angle. Then since $AD=1$ and $AB=7$, $DB=\sqrt{1+49}=\sqrt{50}=5\sqrt{2}$. Since $\angle DCB$ is a right angle, $2CD^2=CD^2+CB^2=DB^2=50\Rightarrow CD=5$. Note the $m\angle ADB=\arctan 7$. Now apply the law of cosines to ΔADC:
(OR Now use Ptolemy's Theorem.)

$$AC^2=AD^2+CD^2-2\cdot AD\cdot CD\cdot\cos(\arctan 7+\tfrac{\pi}{4})$$
$$=1+25-2\cdot1\cdot5\cdot[\cos(\arctan 7)\cos(\tfrac{\pi}{4})-\sin(\arctan 7)\sin(\tfrac{\pi}{4})]$$
$$=26-10\cdot[\tfrac{1}{\sqrt{50}}\cdot\tfrac{1}{\sqrt{2}}-\tfrac{7}{\sqrt{50}}\cdot\tfrac{1}{\sqrt{2}}]=26-10[\tfrac{1}{10}-\tfrac{7}{10}]=32$$

Thus, $AC=\sqrt{32}=4\sqrt{2}$.

39) all $x>0$, $x\neq1$

Note since x appears as the base in the expression $\log_x 5$, we must have $x>0$, $x\neq1$. Furthermore for these possible x, $(\log_3 x)(\log_x 5)=\log_3 5 \Leftrightarrow 3^{(\log_3 x)(\log_x 5)}=3^{\log_3 5} \Leftrightarrow x^{\log_x 5}=5 \Leftrightarrow 5=5$, a statement that is always true. Thus, the equation holds for all $x>0$, $x\neq1$.

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