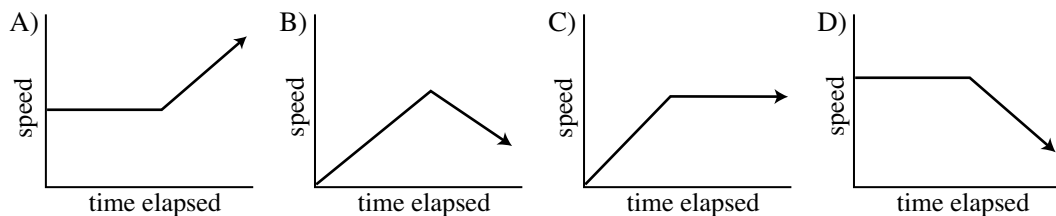
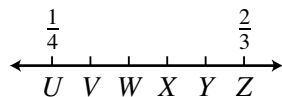


OCTM 2006

- Simplify: $2 + (0 \cdot 0) \cdot 6$.
- Write 2006 in scientific notation.
- Give the more common, but mathematically correct, name for every equilateral quadrilateral.
- Ms. Hiller slowly climbs to the top of a hill at a steady pace and then starts to run downhill on the other side. Which graph best represents this event?



- Find k so that the line through the points $(2006, k)$ and $(-1003, 7)$ is parallel to the line $206y = 2006$.
- Name the largest factor of 2006 that is also a prime number.
- $a, b, c,$ and d are positive integers, and $a > 2.006, b > 2a, c > 3b,$ and $d > 4c$. What is the smallest possible value of d ?



- Points U to Z are evenly spaced on the number line shown. What common fraction corresponds to point Y ?
- The Mathcats scored an average of 56 points per game during their 21-game season. They averaged 66 points per game for each of their 11 home games. How many points per game did they average for each of their away games?
- In a 3-way presidential election, Kenneth received $\frac{5}{6}$ as many votes as Judy, who received 80% as many votes as Linda. If Kenneth received 200 votes, how many votes did Linda receive?
- A fair coin is flipped, and a fair six-sided die is rolled. What is the probability of getting a tail and a prime number?

- Which of the following is an example of an equation of degree 3 which has exactly 3 unique real solutions?
 - $x^3 = x$
 - $x^3 = -x$
 - $x^3 + 4x^2 + 4x = 0$
 - $x^3 - x^2 - x + 1 = 0$
 - There is more than one correct answer.

- Mathematics teachers Sue Lowe and Jeff Reinhardt were grading OCTM State Math Contests. Ms. Lowe said, "If I give you 6 of my tests, then I would have $\frac{2}{3}$ as many as you." Mr. Reinhardt replied, "I would rather give you 10 of my tests; then I will have $\frac{1}{2}$ as many as you." How many tests did each teacher have originally?

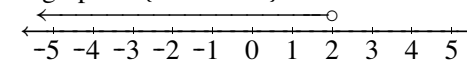
- Find all real values of x such that $200(6 + x) = 0$.
- Let $\begin{bmatrix} 5 \\ y \end{bmatrix} \begin{bmatrix} 3 & 2 & 0 \end{bmatrix} = \begin{bmatrix} a & -b & 2c \\ d+1 & 3e & f-1 \end{bmatrix}$. If $d = -7$, find the exact value of e .
- Superman and Batman are balanced at opposite ends of a teeter-totter (seesaw). Superman weighs 180 pounds and is 9 feet from the fulcrum (pivot). If Batman is 10 feet from the fulcrum, how many pounds does Batman weigh?

- Evaluate $f(0), f(-2),$ and $f(3)$ for the piecewise function $f(x) = \begin{cases} x^3 - 1 & \text{if } x \geq 0 \\ |x - 2| & \text{if } x < 0 \end{cases}$.

- Solve for all real values of x : $\left(\frac{1}{4}\right)^{2x+8} = 8^{2x+5}$.

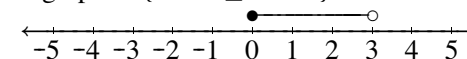
- The sum of the two linear factors of $x^2 - 11x + 24$ is subtracted from the sum of the two linear factors of $x^2 - 10x - 24$. Write the result.

The graph of $\{x : x < 2\}$:



- Shown on the right are two models of number graphs. Using the number line provided on the answer sheet, show the graph of the union of the set $\{x : x^2 - x - 6 < 0\}$ with the set $\{x : x^2 + x - 2 > 0\}$.

The graph of $\{x : 0 \leq x < 3\}$:



- Using U.S. census data, the population of Ohio can be modeled by $P(t) = \frac{12.79}{1 + 2.402e^{-0.0309t}}$, where P is the population in millions, and t is the number of years since the beginning of 1900. Based on this model, during what year did the population of Ohio reach 10 million?

- Given vectors $\mathbf{u}, \mathbf{v},$ and \mathbf{w} , with $\mathbf{u} = 2\mathbf{i} - \mathbf{j}$ and $\mathbf{v} = 3\mathbf{i} + 4\mathbf{j}$, if $\mathbf{v} = \mathbf{w} - \mathbf{u}$, find \mathbf{w} . Express in the form $a\mathbf{i} + b\mathbf{j}$.

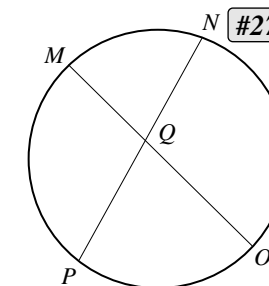
- Given the equations $x = 1 - 2t$ and $y = 2 + t$, eliminate the parameter t and find m and b in the linear equation $y = mx + b$.

- Find all real numbers k such that $|x - k| = 2$ has exactly one solution.

- Let $i = \sqrt{-1}$. Find a and b when $\frac{2006 + i^{2006} - i^{-2006} - 2006i}{2006}$ is expressed in simplest $a + bi$ form.

- A piece of wire 36 inches long is cut into pieces that are soldered together to form the frame of a cube (all edges). Find the volume of the largest cube that can be made with this frame. Be sure to include the correct units.

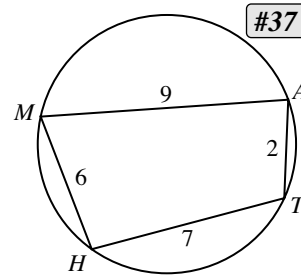
- Given: $m\widehat{MP} = 2x, m\widehat{NO} = 5x + 6,$ and $m\angle MQP = 94^\circ$. If $m\widehat{PO} = 3 \cdot m\widehat{MN}$, find the numerical value of $m\widehat{PO}$.



- Between what two consecutive integers does $\log_{20.06} 2006$ lie?

29. A rectangular swimming pool is twice as long as it is wide, and it has a 10-ft wide concrete border around the entire pool. If the border has an area of 2800 ft^2 , find the perimeter of the pool (in feet).
30. Find the sum of the 3 cube roots of 8.
31. Given the parabola $y = 2x^2 - 8x + 3$. Find the coordinates of the focus. Express in the form (x, y) .
32. In $\triangle ABC$, $\tan A = 1$, $\tan B = \frac{3}{4}$, and $b = 12$. Find the length of side c .
33. Express the series $1 + 4 + 9 + 16 + 25 + 36$ using sigma (Σ) notation.
34. Kan Ga Rue can hop 800 meters in 3 min, 20 secs. How fast is that in kilometers per hour?
35. Five math teachers met for a gab-session at the OCTM Conference in Dayton. Frank Centofani had just won a deck of Trig Flash-Cards in the Vendor Exhibition Area and wished to share with his colleagues.
 Mr. Centofani gave one-half of the cards, plus 2 more cards to Ms. Claudia Hower.
 Ms. Hower then gave one-half of her cards, plus 2 more cards, to Susan Holloway.
 Ms. Holloway then gave one-half of her cards, plus 2 more cards, to Linda Wohlever.
 Ms. Wohlever then gave one-half of her cards, plus 2 more cards, to Fred Warmbrodt.
 Mr. Warmbrodt received 10 cards. How many Flash-Cards did Ms. Hower still have after receiving cards from Mr. Centofani, and then giving some cards to Ms. Holloway?

36. One side of a triangle is 18 centimeters. Inside the triangle, a segment is drawn parallel to the 18-cm side, forming a trapezoid whose area is one-third that of the original triangle. Find the length of this segment (in cm).



37. Given: Inscribed quadrilateral $MATH$ as shown. Give either the name of the person who derived a formula for finding the area of an inscribed quadrilateral (using the semiperimeter), or find the area of quadrilateral $MATH$.

38. Find the distance from point $P(2, -3)$ to the line whose equation is $y = -\frac{3}{5}x + \frac{5}{2}$.

39. In a bag are 3 red, 4 blue, 5 green, and 6 white marbles. If one marble at a time is drawn at random, and not replaced, what is the least number of marbles that must be drawn to ensure having at least 3 marbles of the same color?

40. For the 280-mile trip from Cleveland to Cincinnati for the Monday night Battle-of-Ohio Football Game between the Browns and the Bengals, the Browns bus left at 9 AM and traveled for 150 miles at 57.8 mph. If they stopped for 90 minutes to eat, how fast (to the nearest tenth of a mile per hour) must the bus travel the remaining distance to reach Cincy at 3:15 PM?



THE OHIO COUNCIL OF TEACHERS OF MATHEMATICS

Thirty-third Annual Contest
February 25, 2006

You may write on this test. Please keep this test when you finish.

During the test, each student is permitted to have one handheld calculator, **including** the TI-89, TI-92 and HP95. Calculators with cordless transmission capabilities must be taped over.

On the answer sheet please give your gender (for statistical purposes only). Also we ask for your **home address, phone number and your e-mail address** so that you personally can be quickly notified and invited to attend the OHMIO competition, should you qualify.

Instructions:

- Place each answer on its proper blank on the answer sheet.
- There may be one or more questions which are impossible. For the purpose of this test, write "impossible" or "no solution" or \emptyset or $\{ \}$. **NO CREDIT** given for $\{ \emptyset \}$.
- There may be one or more questions with multiple answers. In such cases, all answers are required unless specified otherwise.
- Write multiple solutions as (e.g.) " $\{2, 3\}$ " **OR** " $x = 2$ or $x = 3$ " **OR** " $2, 3$ ". **NO CREDIT** given for ordered pair form " $(2,3)$ ". **NO CREDIT** given for " $x = 2$ **AND** $x = 3$."
- In problems 1–20, **EXACT ANSWERS IN SIMPLEST FORM** are necessary.
 For example: write " $1 + \sqrt{2}$ " (not 2.414...); write " $\frac{\pi}{4}$ " (not 0.785398...);
 write " $x = 5$ or $x = 1$ " (not 3 ± 2); write " 1 " (not x^0);
 write " $\frac{4}{9}$ " or " $0.\bar{4}$ " (not $\frac{16}{36}$, nor $(\frac{2}{3})^2$, nor 0.4444).
- After problem 20, unless otherwise specified, the answer may be written in **exact decimal, radical or fractional form**, or decimal form **rounded off to four places after the decimal point**.
- The questions are not arranged according to difficulty. (There are some easy problems after number 30. Check it out!)
- Testing time: **60 MINUTES**

Grading:

- Each correct answer counts one point. No partial credit will be given.
- There is no penalty for guessing.