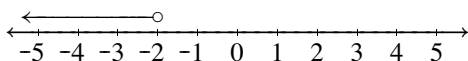


OCTM 2005

- Simplify: $2005 - (2 \cdot 0 + 0^5)$
- Donnie has three more brothers than sisters. How many more brothers than sisters does his sister Marie have?
- What name do we give to a segment that connects any two nonconsecutive vertices of a polygon?
- Solve for x : $20.05x - 20.05 = 2005$.
- Which one of these diagrams, statements, or expressions is NOT equivalent to the other four? Answer with a **capital letter**.

A) $x < -2$ B) $(\infty, -2)$ C) the set of all real numbers less than -2

D) $-3x > 6$ E) 

6. Express as a single fraction in simplest form: $\frac{1}{2 + \frac{3}{4+5}}$

7. Today is Saturday, February 26, 2005. What day of the week will be 2005 days from now? (Clarification: One day from now will be SUNDAY.)

8. Give the range of the function $f(x) = \frac{\sqrt{x+1}}{x-2}$. Answer with a **capital letter**.

A) $(-\infty, \infty)$ B) $(-\infty, 2) \cup (2, \infty)$ C) $[0, \infty)$
D) $[-1, \infty)$ E) $[-1, 2) \cup (2, \infty)$

9. Express in simplest form: $\frac{4^3}{27} + \frac{200}{5} + 2.\bar{9} + (200.5)^0$.

10. Which one of these sets is NOT equivalent to the other four? Answer with a **capital letter**.

A) whole numbers B) positive integers C) counting numbers
D) $\{1, 2, 3, 4, \dots\}$ E) natural numbers

11. How many seconds longer is 3% of an hour than is 20% of a minute?

12. (See figure.) $\triangle ABC$ is isosceles. Find all possible perimeters.

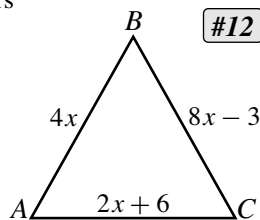
13. Find the y -intercept of the parabola $y + 5 = (x - 4)^2$.

14. Express in **base 10**: 200_{five}

15. Which one of the following polygons **cannot** tessellate the plane? Answer with a **capital letter**.

A) square B) regular pentagon C) obtuse triangle
D) regular hexagon E) kite

16. Write a sine function equation of a sinusoid with amplitude 2, period 2π , which is shifted horizontally $\pi/2$ units, translated one unit up, and passes through the point $(0, 3)$.



17. The ages of ten members of the Boys Club range from 5 to 14, and each boy is a different age. Two boys from each of five families belong. The sums of the ages of each pair of brothers are 12, 15, 19, 24, and 25. One boy is 11. How old is his brother?

18. $y = \frac{x^2 - x - 2}{x^2 - 3x + 2}$. Find the equations of all vertical asymptotes.

19. Consider the following pairs of quantities:

	<u>Column I</u>	<u>Column II</u>	
i.	$(3^3)^2$	$3^{(3^2)}$	
ii.	$2 + 3i$	$3 + 2i$	where $i = \sqrt{-1}$
iii.	supplement of $\angle X$	complement of $\angle X$	where $\angle X$ is an acute angle

For each part, determine the relationship between the two quantities and write for your answer:

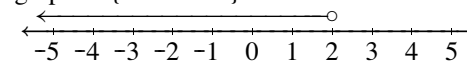
A if the quantity in Column I is greater C if the quantities are equal
B if the quantity in Column II is greater D if none of A, B, or C is true

20. Shown on the right are two models of number graphs. Solve

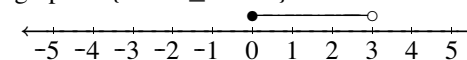
$$3 \geq |x| > -2$$

and graph using the number line provided.

The graph of $\{x : x < 2\}$:



The graph of $\{x : 0 \leq x < 3\}$:



21. Mathematics professor Jomtree invested \$22,055 more at 8% than she did at 4% (both at simple interest). If her total annual return was \$2005, how much was invested at 8%?

22. The formula for the volume of a square pyramid is $V = \frac{1}{3}s^2h$. Solve this formula for h .

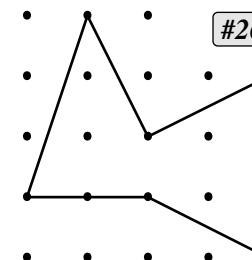
23. The positive integers a, b, c are said to form a Pythagorean triple if $a^2 + b^2 = c^2$. Write a Pythagorean triple in the form a, b, c (with $a < b < c$) such that any of the numbers a or b or c is 2005.

24. Find the coordinates of the reflection image of point $(200, 5)$ when the line of reflection is:

a) the x -axis b) the line $y = x$.

25. Express in simplest form: $\frac{n!(n+1)}{(n-1)!} \cdot \frac{1}{n^2+n}$.

26. (See figure.) The dots on this grid are one centimeter apart, both horizontally and vertically. What is the perimeter of this concave hexagon?



27. Solve for x : $\sin 2005^\circ + \cos x^\circ = \tan 2005^\circ$, where $0 \leq x < 360$.

28. The line $y = 3kx + 4k$ is perpendicular to the line $2x - y = 5$. Find k .

29. A sphere of radius 7 cm is intersected by a plane passing 6 cm from the center. Find the area of the circle of intersection.

30. This stem-and-leaf plot (also called a stemplot) represents scores on a 100-point mathematics test. Let x be the first (lower) quartile and y be the third (upper) quartile. Compute $x - y$.
- | | |
|---|-----------|
| 6 | 3 3 8 |
| 7 | 2 2 2 7 |
| 8 | 3 4 8 8 9 |
| 9 | 1 2 3 |
31. Solve for x : $4 \log_x 2 + \log_2 x = 5$.
32. Define the operation \otimes by: $a \otimes b = a^2 + 4a - 4$, for all real numbers a and b . Find the value(s) of a for which $a \otimes 2 = a$.
33. A sphere is inscribed in a cylinder, being tangent to the sides and to the top and bottom. The volume of the cylinder is 2005 cm^3 . Find the surface area of the sphere.
34. Given the conic section $y^2 - 12x - 2y + 25 = 0$, find the length of the *latus rectum*, or focal chord (the segment through the focus perpendicular to the axis of symmetry).
35. At the OCTM Annual Conference, there were 3 red, 3 white, and 3 blue marbles in a bag. Teacher Rachel Temple randomly pulled out a marble and replaced it, then teacher Jan Von Spiegel randomly pulled out a marble and replaced it, and then teacher Anne Newcomb randomly pulled out a marble. What is the probability that *exactly* two of the marbles were the same color?
36. For what positive value(s) of x is $x + 2$ the reciprocal of $x - 2$?
37. We want to use *mathematical induction* to prove that the minimum number of moves in the “Towers of Hanoi” problem is $2^n - 1$. Which of the following will be our first step? (Answer with a **capital letter**.)
- Assume that $2^n - 1$ is NOT the correct minimum number of moves.
 - Show that $2^1 - 1 = 1$ (the number of moves needed to move one ring to a new peg).
 - Multiply $(2^n - 1)$ by $(2^n + 1)$.
 - Let $n = 1,000,000$, and let $n + 1 = 1,000,001$.
 - Show that 2^k being true implies that 2^{k+1} is true.
38. The Gateway Arch in St. Louis is an inverted “hanging chain,” which is a “curve assumed by a loose string hung freely from two fixed points.” This special curve occupied the mathematical community for decades before the invention of the calculus. Mathematicians studying this curve included the Bernoulli brothers, Christian Huygens, and Gottfried Leibniz. The equation of this curve is given as $y = \frac{e^{ax} + e^{-ax}}{2a}$, where a is a constant. Write a mathematical name given to this curve.
39. The mean of a, b, c, d , and e is m , while the mean of f, g , and h is n . Find the mean of a, b, c, d, e, f, g , and h in terms of m and n .
40. From Ayersville, mathematics teacher Rosie Kieffer drove 110 miles to Butler High School (Vandalia) to drop off some old OCTM contests to teacher Brian Tregoning, taking her 2 hours, 20 minutes. Then she drove 215 miles to Fitch High School (Austintown) to deliver some old contests to teacher Tom Reardon, taking her $3 \frac{2}{3}$ hours. At what rate must she drive back to Ayersville (170 miles) to average 55 mph for the entire trip?



THE OHIO COUNCIL OF TEACHERS OF MATHEMATICS

Thirty-second Annual Contest February 26, 2005

You may write on this test. Please keep this test when you finish.

During the test, each student is permitted to have one handheld calculator, **including** the TI-89, TI-92 and HP95. Calculators with cordless transmission capabilities must be taped over.

On the answer sheet please give your gender (for statistical purposes only). Also we ask for your **home address, phone number and your e-mail address** so that you personally can be quickly notified and invited to attend the OHMIO competition, should you qualify.

Instructions:

- Place each answer on its proper blank on the answer sheet.
- Write the empty set as \emptyset or $\{ \}$. **NO CREDIT** given for $\{\emptyset\}$.
- There may be one or more questions with multiple answers. In such cases, all answers are required unless specified otherwise.
- Write multiple solutions as (e.g.) “{2, 3}” **OR** “ $x = 2$ or $x = 3$ ” **OR** “2, 3”. **NO CREDIT** given for ordered pair form “(2,3)”. **NO CREDIT** given for “ $x = 2$ **AND** $x = 3$.”
- In problems 1–20, **EXACT ANSWERS IN SIMPLEST FORM** are necessary.
For example: write “ $1 + \sqrt{2}$ ” (not 2.414...); write “ $\pi/4$ ” (not 0.785398...);
write “ $x = 5$ or $x = 1$ ” (not 3 ± 2); write “1” (not x^0);
write “ $4/9$ ” or “ $0.\bar{4}$ ” (not $16/36$, nor $(2/3)^2$, nor 0.4444).
- After problem 20, unless otherwise specified, the answer may be written in **exact decimal, radical or fractional form**, or decimal form **rounded off to four places after the decimal point**.
- The questions are not arranged according to difficulty. (There are some easy problems after number 30. Check it out!)
- Testing time: **60 MINUTES**

Grading:

- Each correct answer counts one point. No partial credit will be given.
- There is no penalty for guessing.
- There may be one or more questions which are “impossible.” In such an event, write “impossible” or “no solution.” **NO CREDIT** given for \emptyset .