

OCTM 2004

- Simplify: $(2 + 1 \cdot 0)^4$
- Write the inverse of "If Reds, then not Indians."
- List all of the following numbers that are **not** whole numbers:
 -3 , $\sqrt{16 + 4 + 1}$, 0 , $\sqrt{2004 + 21}$, $2003.\overline{9}$
- Name the smallest square integer that has exactly 7 distinct positive factors.
- Given point $A(2, 0)$ and point $B(0, 4)$, find the midpoint of \overline{AB} .
- Answer each of the following statements with **A (Always)**, **S (Sometimes)**, or **N (Never)**. (All answers must be correct to receive credit.)
 - If \overleftrightarrow{PQ} and \overleftrightarrow{RS} are skew, then \overleftrightarrow{PQ} **A-S-N** intersects \overleftrightarrow{RS} .
 - The diagonals of a rhombus are **A-S-N** perpendicular.
 - The consecutive angles of a parallelogram are **A-S-N** congruent.
- If 8 is 2 less than $2004a + 3$, find the value of $2004a - 3$.
- Give the domain for the function $f(x) = \frac{x^2 - 4}{2004}$.
- Given: $f(x) = 4x - 2004$, and $g(x) = 5^x$. Find $f(g(4))$.
- The graph of the line $y = 2x - 5$ contains one side of a square. Which one of the following lines **could** contain another side of the square?
 A) $y = 5x - 2$ B) $y = -2x + 5$ C) $y = -0.5x - 5$ D) $y = 0.5x + 5$
- In a 3-way OCTM presidential election, Annie received 35% of the votes, George received 185 votes, and Dan received $\frac{2}{5}$ of the votes.
 - Who was elected President?
 - How many votes were received by the winner?
- A pair of dice is thrown. Mathematics teacher Henry Nixt wins if the sum of the dice is 1, 2, or 3. Mathematics teacher Dennis Kushlak wins if the sum of the dice is 4, 5, or 6. Otherwise mathematics teacher Doug Knapke wins. Find the **odds** that Mr. Knapke wins. Express as a rational number a/b in simplest form.
- What is the smallest positive integral value of k for which $35x^2 + kx + 18 = 0$ will have two distinct real number solutions?
- In a class of 100 students, 85 liked mathematics, and 76 liked science. If 66 students liked both mathematics and science, how many students liked neither mathematics nor science?
- If $2x^2 - 5x$ is subtracted from $(ax + b)(2x + 1)$, the result is $4x^2 - 4$. Find a and b .
- In a ten-team league, each team plays each other team twice. How many league games are played?

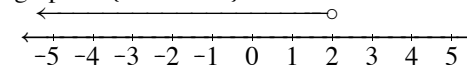
17. Given: $\frac{2}{3} = \frac{3x - 2}{4y - 3}$. Find the ratio of x to y in simplest form.

18. On the top of a right rectangular prism (a box) is a small rectangular hole, measuring 3 cm by 4 cm. If the box measures 12 cm by 20 cm by 24 cm, how much cardboard was used to construct this box? Be sure to include the correct units.

19. Suppose $\log_{2004} 20 - \log_{2004}(0 + 4) = \log_{2004} x$. Find x .

20. Shown on the right are two models of number graphs. Solve

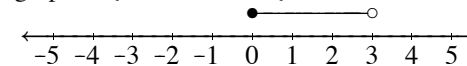
The graph of $\{x : x < 2\}$:



$$x^2 - x - 2 \geq |x - 2|$$

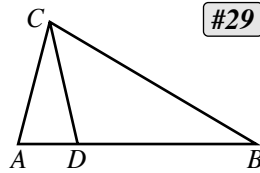
and graph using the number line provided.

The graph of $\{x : 0 \leq x < 3\}$:



- The recent economic downturn has taken a toll on my investments. Investment A had a percent decrease of 30% and is now worth \$200,400. Investment B had a percent decrease of 20% and is now worth \$20,040. Investment C had a percent decrease of 10% and is now worth \$2004. What is the **total** percent decrease of these investments? (Express your answer to the **nearest tenth** of one percent.)
- Mathematics teachers Alexa Collins, Kevin Horstman, and Bonnie Cook graded 112 OCTM Mathematics Contests. Each teacher graded a number of contests that was either a perfect square or a perfect cube, but **not both**. If Ms. Collins graded the most, how many might she have graded? (Give all possible answers for Ms. Collins.)
- The volume of a cylinder whose diameter is equal to its height is 2004 cu. cm. Find the volume (in cu. cm) of the **inscribed sphere**.
- In a bag are 3 red, 4 blue, 5 brown, and 6 white marbles. If one marble at a time is drawn at random, and not replaced, what is the **fewest** number of marbles that must be drawn to ensure having at least 3 marbles of the same color?
- Given: Cubic equation $x^3 + ax^2 + bx + c = 0$, with real coefficients. The sum of the roots is 3, the product of the roots is -24 , and one root is twice another root. Find a , b , and c .
- The air speed (speed in still air) of a light plane is 200 km/hr, and its heading is 140° . A wind of 30 km/hr is blowing from 40° . Find the plane's ground speed—speed relative to the ground—to the nearest whole km/hr.
- Solve for all real values of x : $2^{3x} = 2004$.
- If $x + \frac{1}{x} = 3$, then $x^2 + \frac{1}{x^2} = ?$

29. See figure. Given $\triangle ABC$ as shown, with point D on \overline{AB} , and $AC = CD$, $AD = 1$, $DB = 3$, and $BC = 4$. Find AC .

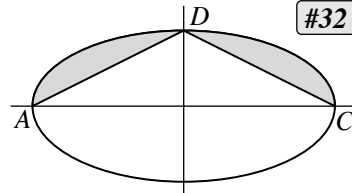


30. Given: $\sum_{k=17}^n [(k+1)^2] - 6 = 2004$. Find n .

31. Given: the determinant equation $\begin{vmatrix} a & 1 \\ 2 & 3 \end{vmatrix} \cdot \begin{vmatrix} 0 & 1 \\ 2 & b \end{vmatrix} = \begin{vmatrix} 2 & 2004 \\ 6 & 2 \end{vmatrix}$.

Using numerical values for a and b , find any ordered pair (a, b) that satisfies this equation.

32. See figure. Given ellipse $\frac{x^2}{100} + \frac{y^2}{25} = 1$, with \overline{DA} and \overline{DC} as shown. Find the area of the shaded region.



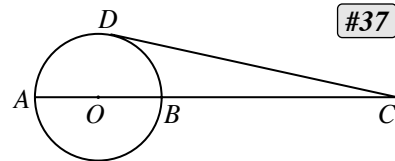
33. Let $i = \sqrt{-1}$. Find the smallest integer $n > 1$ such that $i^{2004} + i^n + i^{3n} = (2^0 + 0^4)$.

34. Let $345_6 = 1022_b$. Find b .

35. Parents Mr. and Mrs. Zeta want to name their new baby so that its monogram (first, middle, last initials) will be in alphabetical order with no letter repeated. How many such monograms are possible?

36. $\sin \theta \cdot \cos \theta \cdot \tan \theta \cdot \cot \theta \cdot \sec \theta = f(\theta)$, where f is one of the trigonometric functions. Which one?

37. See figure. \overleftrightarrow{CD} is tangent to Circle O at point D . \overline{BC} is 4 times the length of the radius of Circle O . If the length of \overline{CD} is 48, find the length of \overline{AB} .



38. This mathematician was the most prolific writer of mathematics. His/her name is attached to every branch of mathematics. Although blind in the last 17 years of her/his life, this was one of his/her most productive periods. Though far too numerous to list, here are a few of this mathematician's contributions: using $f(x)$ for functional notation; using the variables a, b, c, \dots ; using the symbols e, i , and \sum ; and the equation " $e^{i\pi} = -1$ ". Who is this mathematician, born in Switzerland in 1707?

39. Several rectangles with integral sides have an area of 2004 square feet. Of these rectangles, find the rectangle that has the smallest perimeter. How many feet are in that perimeter?

40. Mathematics teacher Cindy Depoe can canoe in still water at 5 mph, while mathematics teacher David Abineri can canoe in still water at 6 mph. The rate of the current in the Great Miami River is 3 mph. Mr. Abineri left point A at noon and canoed upstream on the Great Miami River west of Cincinnati. He reached point B at 2 P.M. Ms. Depoe immediately left point B and canoed downstream to point A. At what time (P.M.) did she arrive at point A?



THE OHIO COUNCIL OF TEACHERS OF MATHEMATICS

Thirty-first Annual Contest

February 28, 2004

You may write on this test. Please keep this test when you finish.

During the test, each student is permitted to have one handheld calculator, **including** the TI-89, TI-92 and HP95. Calculators with cordless transmission capabilities must be taped over.

On the answer sheet please give your gender (for statistical purposes only). Also we ask for your **home address, phone number and your e-mail address** so that you personally can be quickly notified and invited to attend the OHMIO competition, should you qualify for such.

Instructions:

- Place each answer on its proper blank on the answer sheet.
- Write the empty set as \emptyset or $\{ \}$. **NO CREDIT** given for $\{\emptyset\}$.
- Write multiple solutions as (e.g.) " $\{2, 3\}$ " **OR** " $x = 2$ or $x = 3$ " **OR** " $2, 3$ ". **NO CREDIT** given for ordered pair form " $(2,3)$ ". **NO CREDIT** given for " $x = 2$ **and** $x = 3$."
- In problems 1–20, **EXACT ANSWERS IN SIMPLEST FORM** are necessary. For example: write " $1 + \sqrt{2}$ " (not 2.414...); write " $\pi/4$ " (not 0.785398...); write " $x = 5$ or $x = 1$ " (not 3 ± 2); write " 1 " (not x^0); write " $4/9$ " or " $0.\overline{4}$ " (not $16/36$, nor $(2/3)^2$, nor 0.4444).
- After problem 20, unless otherwise specified, the answer may be written in **exact decimal, radical or fractional form**, or decimal form **rounded off to four places after the decimal point**.
- The questions are not arranged according to difficulty. (There are some easy problems after number 30. Check it out!)
- Testing time: **60 MINUTES**

Grading:

- Each correct answer counts one point. No partial credit will be given.
- There is no penalty for guessing.
- There may be one or more questions which are "impossible." In such an event, write "impossible" or "no solution." **NO CREDIT** given for \emptyset .