OCTM 2003

1. Evaluate: $(2^0 + 0)^3$

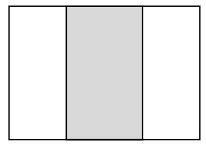
2. Find the largest square integer **LESS THAN** 2003.

3. Find a value of x such that $|x| + 3 = 2$.

4. Give the letter of the one following number that is **NOT** a rational number.
   A) 3.121221222122221 . . .   B) 3.1   C) $3.\overline{1}$ D) $-\frac{2}{13}$ E) 0

5. Solve for x: $200x + 3x = 2003 + 3x - 203$.

#6



6. See figure. Two 7×7 squares overlap to form a 7×10 rectangle. Find the area of the shaded region.

7. Find the sum of the reciprocals of all the positive integral factors of 6.

8. Give the letter of the expression that is **NOT** an example of the commutative property.
   J) $(7+3)2 = 2(7+3)$
   K) $(7+3)2 = (3+7)2$
   L) $(cd)(a+b) = c[d(a+b)]$
   M) $(cd)(a+b) = (dc)(a+b)$

9. What is the 2003rd letter in the repeating expression QWERTYQWERTYQWERTY. . . . . .?

10. There are 64 1×1 squares on an 8×8 checkerboard. How many 2×2 squares are on an 8×8 checkerboard?

11. The sum of 11 different positive odd numbers is 2003. Find the greatest number that can be used as one of these numbers.

12. The mean of 4 numbers is 2003. The mean of 3 of those numbers is 2000. What is the 4th number?

13. Two sides of an isosceles triangle have lengths 4 and 5. Give all possibilities for the perimeter of this triangle.

14. What is the probability that all four of the last four digits of a telephone number are prime? Express as a fraction in lowest terms.

15. Find the number of positive integral factors of $2^3 \times 3^4 \times 2003$.

16. A car travels 2003 miles in $\underline{t}$ hours. How far will it travel in $\underline{h}$ hours at the same average rate?

17. The bases of an isosceles trapezoid have lengths $(2m + n)$ and $(2m - n)$. One base angle is $45°$. Find the area of the trapezoid in terms of m and n.

18. Write the Pythagorean Triple that is made up of the smallest possible composite numbers.

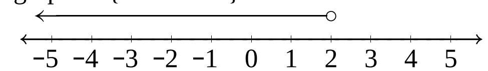
19. Given: $x > y$ and $y > z$. For each of the following statements, write "+" if the statement is **true**, and write "0" if the statement is **false**.
    a) $x > z$ b) $x + w > y + w$ c) $w - y > w - z$

20. Shown on the right are two models of number graphs. Solve

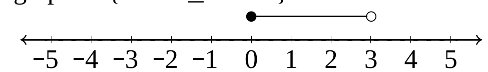
$$|2x + 3| + |x - 6| > 9$$

for all real numbers x and graph your answers on the number line provided.

The graph of $\{x : x < 2\}$:



The graph of $\{x : 0 \le x < 3\}$:



21. Mrs. Miser invested a certain amount of money at 3% compounded yearly. At the end of 20 years, the investment was worth \$2003. To the nearest dollar, how much did she invest?

22. Yesterday 3 candy bars sold for \$3.30. Today they are on sale, and 4 candy bars can be bought for \$3.00. Find the percent of decrease (to the nearest tenth of one percent) in the price per candy bar from yesterday to today.

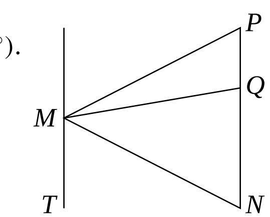
23. $\dfrac{\cot\theta - 1}{1 - \tan\theta}$ is equal to which one of the six trigonometric functions of θ?

24. What is the reflection of the point $(3, 1)$ over the line $y = 2x$?

25. Find the distance between the polar coordinate points $(2, 30°)$ and $(4, 150°)$.

26. Find the inverse function $f^{-1}(x)$ of the function $f(x) = 2x + 5$.

#27



27. See figure. In isosceles $\triangle MNP$, $MN = MP$, $NQ = 2PQ$, and $\overline{MT} \parallel \overline{NP}$. If $m\angle MPN = 50°$, find $m\angle TMP$.

28. For the function $f(x) = \dfrac{x^2 - 6x + 8}{2x^2 - 5x + 3}$, let $y = a$ be the horizontal asymptote, and $x = b$ and $x = c$ be the vertical asymptotes. Compute the value of $(b + c) \div a$.

29. These five mathematics teachers attended the Ohio Council of Teachers of Mathematics Annual Conference. Robert Plummer attended 13 sessions, Kathy Woolison attended 10 sessions, Adrian Mangino attended 11 sessions, and Andrew Sleek attended twice as many sessions as did Karla Barnes. If these five teachers attended 55 sessions altogether, how many sessions did Andrew Sleek attend?

30. Given the table on the right, write the inverse element of a.

#30

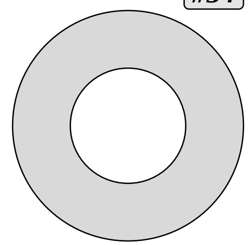
□	*a*	*b*	*c*	*d*
a	*c*	*d*	*a*	*b*
b	*d*	*a*	*b*	*c*
c	*a*	*b*	*c*	*d*
d	*b*	*c*	*d*	*a*

31. At a summer mathematics workshop, teachers were asked to draw a polygon and its diagonals. Barb Christman drew a triangle, Harvey Cohen drew a quadrilateral, Jerry Bunn drew a pentagon, Mark Meuser drew an octagon, and Ken Huffman drew a polygon. Mr. Meuser's polygon had the number of diagonals that was 12 more than one-half the sum of all the other diagonals. How many SIDES did Mr. Huffman's polygon have?

32. If $\begin{bmatrix} a & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 2 & b \end{bmatrix} = \begin{bmatrix} 2 & 203 \\ 6 & 11 \end{bmatrix}$, solve for a and b.

33. A pool receives its water from two inlet pipes and is drained through a single outlet pipe. Each inlet pipe, when used alone, can fill the pool in 20 minutes, and the outlet pipe can drain the pool in 12 minutes. If the three pipes are opened at the same time, how many minutes will it take to fill the pool?

#34



34. See figure. The area of the smaller (inner) circle of a ring is 16π square units, and the circumference of the larger (outer) circle is 16π units. Find the number of square units in the area of the ring.

35. If a and b are whole numbers with $4 < b < 2003$ and $17 < a < 76$, what is the greatest possible value for $(a + b)/b$?

36. Given points $A(-7, 4)$ and $B(1, 6)$, find the length of the segment joining the midpoint of $\overline{AB}$ and the center of the circle $x^2 + y^2 - 8y + 10x = 25$.

37. Solve for x: $(\log_x 2x)(\log_{10} x) = 3$.

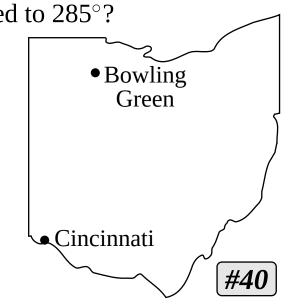
38. Born about 630 BC in Miletus, Asia Minor (now Turkey), this person seems to be the first known Greek philosopher, scientist, and mathematician, although his occupation was that of a merchant. He first went to Egypt, where he measured the heights of pyramids by observing shadows. He then introduced the study of geometry into Greece. It is said that demonstrative geometry (proofs) began with this man, one of the "seven wise men" of antiquity. He is credited with using logic to prove the following:

    1) A circle is bisected by any diameter.
    2) The base angles of an isosceles triangle are congruent.
    3) An angle inscribed in a semicircle is a right angle.

    And there is the story of this man's mule, which when transporting salt, found that by rolling over in the stream he could dissolve the contents of his load and thus travel more lightly. This man broke the mule of this troublesome habit by loading him with sponges. Who is this earliest geometer?

39. The pressure of a gas varies directly as its absolute temperature and inversely as its volume. A gas has volume of 500 cubic centimeters when under a pressure of 30 centimeters of mercury and at an absolute temperature of 300°. What pressure in centimeters would be exerted by the gas if the volume rose to 570 cubic centimeters and its absolute temperature dropped to 285°?

40. Contest Director Charlene (Charlie) drove 180 miles from Cincinnati at a constant rate to meet with Site Director Barbara in Bowling Green. If Charlie had driven at a rate that was 6 miles per hour slower, it would have taken her 20 more minutes to reach Barbara. What was Charlie's original speed in miles per hour?



#40

THE OHIO COUNCIL OF TEACHERS OF MATHEMATICS

Thirtieth Annual Contest

February 22, 2003

During the test, each student is permitted to have one handheld calculator, **including** the TI-89, TI-92 and HP95. Calculators with cordless transmission capabilities must be taped over.

On the answer sheet please give your gender (for statistical purposes only). Also we ask for your **home address, phone number and your e-mail address** so that you personally can be quickly notified and invited to attend the OHMIO competition, should you qualify for such.

Instructions:

- Place each answer on its proper blank on the answer sheet.
- Write the empty set as $\varnothing$ or { }. **NO CREDIT** given for $\{\varnothing\}$.
- Write multiple solutions as (e.g.) "{2, 3}" **OR** "$x = 2$ or $x = 3$" **OR** "2, 3".
  NO CREDIT given for ordered pair form "(2,3)".
  NO CREDIT given for "$x = 2$ **and** $x = 3$."
- In problems 1–20, **EXACT ANSWERS IN SIMPLEST FORM** are necessary.
  For example: write "$1 + \sqrt{2}$" (not 2.414...); write "$\pi/4$" (not 0.785398...);
  write "$x = 5$ or $x = 1$" (not 3 ± 2); write "1" (not x^0);
  write "4/9" or "$0.\overline{4}$" (not 16/36, nor $(2/3)^2$, nor 0.4444).
- After problem 20, unless otherwise specified, the answer may be written in **exact decimal, radical or fractional form,** or decimal form **rounded off to four places after the decimal point**.
- The questions are not arranged according to difficulty. (There are some easy problems after number 30. Check it out!)
- Testing time: **60 MINUTES**

Grading:

- Each correct answer counts one point. No partial credit will be given.
- There is no penalty for guessing.
- There may be one or more questions which are "impossible." In such an event, write "impossible" or "no solution." **NO CREDIT** given for $\varnothing$.