OCTM Math Contest 2002

THE OHIO COUNCIL OF TEACHERS OF MATHEMATICS
Twenty-Ninth Annual Contest
February 23, 2002

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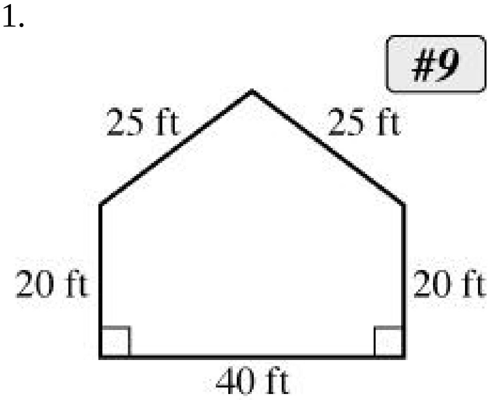
1. Simplify: $(2 + 0)^{(0 + 2)}$.
2. A palindrome is a word or a number that reads the same forward and backward. This year 2002 is a palindromic year. Find the next palindromic year.
3. Rosy Teacher has 26 children in her morning kindergarten class and 24 children in her afternoon kindergarten class. What is the minimum number of desks, including her own, that she must have in the classroom so that each person in there at any one time has a desk?
4. Find all real values of x such that $\sqrt{2+0+0+2} = x^2$.
5. Mathematics teacher and farmer Tom Elsass's red barn has two and one-half times as many hay bales as his white barn has. If the two barns contain a total of 2002 bales, how many bales are in the red barn?
6. Two positive integers are chosen randomly. What is the probability that their product is even? Express as a ***percent***.
7. If you arrange the consecutive integers 10, 11, 12, ..., 18 to form a 3×3 magic square, what is the ***sum*** of the integers in any one row, column, or diagonal?
8. Given that $A \bigstar B = A + 2(B - 5)$, find m such that $5 \bigstar m = -11$.



9. Van Varnish is going to paint the end of the house as shown. If one gallon of paint covers 250 ft^2, and paint only comes in gallons, how many gallons must Van go buy in order to paint the end of the house?
10. Solve for all ***negative integral*** values of x: $(x - 1)(x - 3) = 15$.
11. The following "proof" establishes this statement: <u>If $a = b$, $a > 0$, and $b > 0$, then $1 = 2$.</u> Obviously something is wrong. Write the ***number*** of the step that contains the error.

Proof:

1. $a > 0, b > 0$	1. Given
2. $a = b$	2. Given
3. $ab = b^2$	3. Multiply both sides by b
4. $ab - a^2 = b^2 - a^2$	4. Subtract a^2 from both sides
5. $a(b - a) = (b + a)(b - a)$	5. Factor
6. $a = (b + a)$	6. Divide both sides by $(b - a)$
7. $a = a + a$	7. Substitute a for b

8. $a = 2a$	8. Simplify
9. $1 = 2$	9. Divide both sides by a

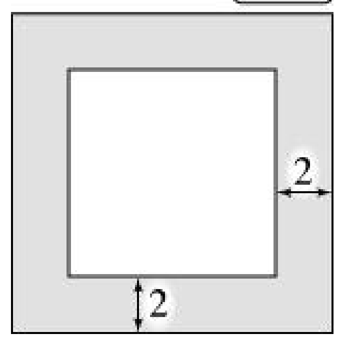
12. A regular work week consists of 40 hours, and overtime pays 1 1⁄2 times regular wages. Last week librarian Pearl Book earned $920.92 (before deductions) by working 44 hours. What is Pearl's regular hourly wage?

13. During the year, maternity nurse Debbie Dewgood treated 2002 patients, who were mothers and newborns. If the only multiple births were 10 sets of twins and 3 sets of triplets, how many ***mothers*** were there?

14. In a 3-way OCTM presidential election, Linda received 350 votes, Bonnie received 250 votes, and Isaac received 25% of the votes. What exact percentage of the votes did Bonnie receive?

15. In a rectangular coordinate plane, any circle that passes through (1, -2) and (3, 4) cannot also pass through (x, 2002). Find x.

#16



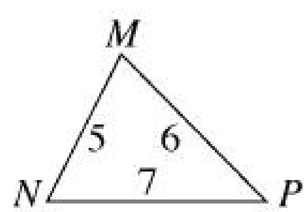
16. See figure. If the uniform distance between the two squares is 2, and the area of the shaded region is 56, find the length of one side of the smaller square.

17. In a 30°–60°–90° triangle, the longest side and the shortest side differ in length by 2002 units. What is the length of the longest side?

18. Sets A, B, and C contain the following mathematics teachers:
 A = { Roseann Barlow, Patricia Benedict, Susan Cunningham, Leah Evans }
 B = { Leah Evans, Pam Riffle, Betty Mills }
 C = { Pam Riffle, Ron Engel, Dennis Gwirtz, Donn Manker, Bill Scott }
 Given that the universal set U consists of all ten of these mathematics teachers, ***how many*** mathematics teachers are in the set $\overline{(A \cup B)} \cap C$? Here $\overline{X}$ denotes the complement of the set X.

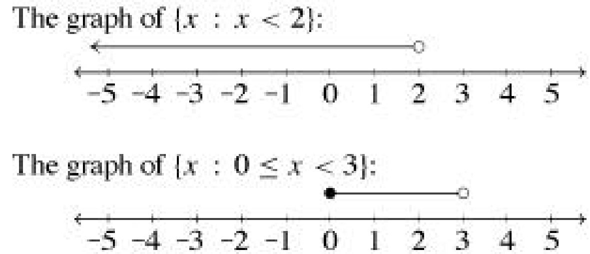
19. Consider the following three pairs of quantities:

	Column A	Column B	
i.	$m\angle N$	$m\angle P$	in the triangle
ii.	sin 30°	cos(π/3)	
iii.	x	y	where $x > 0$ and $y = x^2 - x$



For each part, determine the relationship between the two quantities and write for your answer:

A	if the quantity in Column A is always greater
B	if the quantity in Column B is always greater
C	if the quantities are always equal
D	if none of A, B, or C is true



20. Shown on the right are two models of number graphs. Solve

$$2001(x - 1) < 2002(x - 2) + 2002$$

for all real numbers x and graph your answer on the number line provided. Please mark your answer ABOVE the line.

21. In one year an account of \$2002 decreased in value by 10%. In the following year the account decreased by 10%. In the third year, the account returned to its original value of \$2002. To the nearest ***tenth*** of one percent, by how much did the account increase during the third year?

22. The sum of all but one of the measures of the interior angles of a convex polygon is 2002°. What is the measure (in degrees) of that one angle?

23. Solve for x: $\log(3x + 10) - \log(x + 2) = \log 2x$.

24. Quarterback Q.B. Sax has completed 132 passes. If he completes his next 12 passes, his completion rate will be exactly 80%. What is his current completion rate (to the nearest ***tenth*** of one percent)?

25. The mathematics curriculum review committee has five mathematics teachers as members: Scott McCormick, Ken Rahn, Jennifer Walls, Pam Wendel, and Wendy Williams. Three of them are randomly selected to be on the executive board. What is the probability that both Jennifer and Wendy are selected? Express as a fraction in lowest terms.

26. Solve for all real values of x: $(x^2 - 5x + 5)^{(x^2 - 9x + 20)} = 1$.

27. Find ***and simplify*** the tenth term of $(a + b)^{14}$ if arranged in decreasing powers of a.

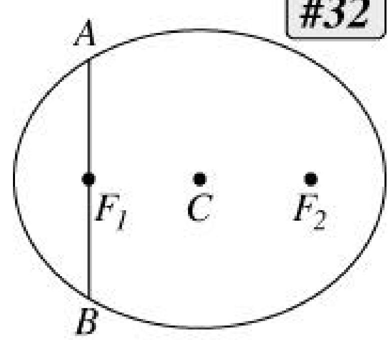
28. Carey drew a circle with center C and diameter 5 cm long. From an external point, both a tangent and a secant are drawn to the circle, the secant passing through C. If the length of the tangent is 3.5 cm more than the length of the radius of the circle, how far (in cm) from C is the external point?

29. If $3x^3 - 8x^2 + 7$ is written in the form $a(x - 2)^3 + b(x - 2)^2 + c(x - 2) + d$, find the numerical value of the sum $a + b + c + d$.

30. At 3:00 the hands of a clock form an angle of 90°. At what ***exact*** time between 3:00 and 4:00 do the hands of the clock again form a 90° angle?

31. $121_{\text{ten}} - 121_{\text{eight}} = N_{\text{five}}$. Find N.

#32



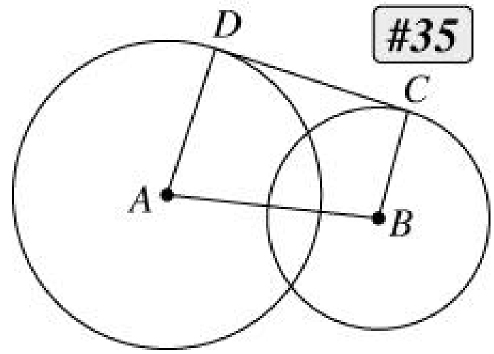
32. The figure shows an ellipse with center C, foci F_1 and F_2, major axis length 20, and minor axis length 16. Find the length of the *latus rectum*, i.e., $\overline{AB}$.

33. Mathematics experts Don Gerke, Darryl Nester, Harlan Basinger, Tena Roepke, and Pat Johnson ***each*** wrote a minimum of 3 problems for this 40-problem OCTM Contest, and each one wrote a ***different*** number of problems. If Darryl wrote the most, which was twice the number written by Tena, the second most, what is the ***maximum*** number of

problems that Darryl could have written?

34. A tennis ball (sphere) is inscribed in a one-ball tennis can (cylinder). Find the ratio of the ball's surface area to the can's ***lateral*** surface area. Express your answer as a ratio *a* : *b* in lowest terms.

35. See figure. The radius of circle *A* is 12; the radius of circle *B* is 8. If the centers are 16 units apart, what is the area of quadrilateral *ABCD*, where $\overline{CD}$ is a common external tangent?



36. If tan *x* + cot *x* = 25/16, find the value of $\dfrac{1}{\tan x} + \dfrac{1}{\cot x}$.

37. There are 200 fish in an aquarium. 99% of them are guppies. How many guppies must be removed to reduce the percentage of guppies to 98%?

38. I was a 17th century French mathematician who had a lifelong habit of lying in bed until late in the morning (or all day). According to legend, I noticed a fly (or spider) on the ceiling and realized that I could determine its position by assigning a horizontal and a vertical distance to it. Fermat and I are usually given credit for founding analytic geometry (where position is located by points on the coordinate plane), and therefore he and I are often considered to be the first modern mathematicians. I am also well known for my rule of signs, used in determining the upper bound to the number of positive zeros and to the number of negative zeros of a polynomial. On top of all this, I am often regarded as the founder of modern philosophy ("I think, therefore I am"). Who am I?

39. List all the ***positive integral factors*** of 2002 that are also ***prime numbers***.

40. Two trains on the same track between Six Flags and Kings Island are approaching each other at the rate of 70 mph and 80 mph. These two sites are 250 miles apart, with a bee flying back and forth between the two engines at a rate of 120 mph. If the bee and the trains all started at the same time, how many miles does the bee fly before the trains meet?