

OCTM Math Contest 2001

THE OHIO COUNCIL OF TEACHERS OF MATHEMATICS
 Twenty-Eighth Annual Contest
 February 24, 2001

Instructions

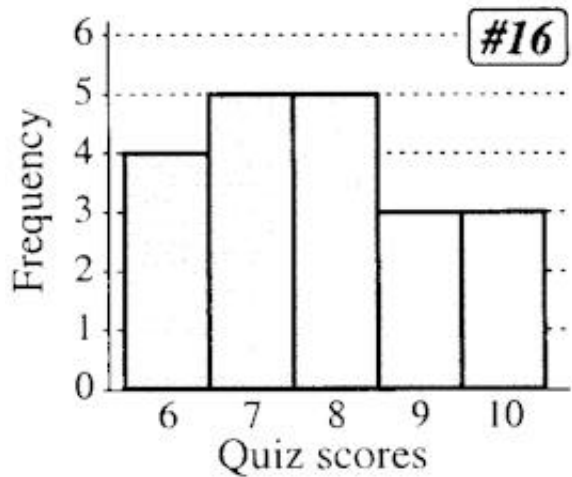
1. Simplify: $2^0 + 0^1$.
2. Solve for all real values of x : $200x + 1 = 2001$.
3. What is the first Fibonacci number, greater than 1, that is a perfect square?
4. Ms. Munny has \$20.01, consisting of one penny and an equal number of nickels, dimes, and quarters. How many coins does she have altogether?
5. If the time right now is 10:00 AM, what time will it be (including AM or PM) 2001 hours from now?
6. Find the smallest integer n for which $\frac{2001}{n+3}$ is an integer.
7. If 2 distinct numbers are randomly drawn from the first ten positive integers (without replacement), what is the probability that both numbers will be odd? Express as a fraction a/b in lowest terms.
8. Find a point on the line $y = x$ that satisfies the equation $3x - 2y = 2001$. Express as an ordered pair (x, y) .
9. $a \cup b = b^2 - a^2$, and $g(a) = a + 2$. Find $3 \cup g(3)$.
10. See figure. With segment lengths as shown, the area of rectangle $GEOM$ is equal to the area of rectangle $MATH$. Find the length of segment \overline{AG} .
11. What is the largest natural number less than 2001 that is divisible by 3 and the sum of whose digits is divisible by 5?
12. I have two brothers and two sisters. My mother's parents have 12 grandchildren, and my father's parents have 15 grandchildren. If no divorces or remarriages have occurred, how many first cousins do I have?
13. Write the simplest quadratic equation in the form $ax^2 + bx + c = 0$ (where a, b, c are integers) that has roots $-4/3$ and 0.6 .
14. Answer each of the following statements with **A (Always)**, **S (Sometimes)**, or **N (Never)**.

[All answers must be correct to receive credit.]

- a) An equilateral triangle is **A-S-N** acute.
- b) An isosceles triangle is **A-S-N** scalene.
- c) A scalene triangle is **A-S-N** a right triangle.

15. The diagonal of a rectangular solid has length $\sqrt{2001}$. The length of the solid is 44; its height is 1. Find its width.

16. See figure. Shown is a histogram of a set of quiz scores. Let x = the mean and let y = the median. Write the value of $x - y$.



17. If $x + y = 7$, and $x^2 - y^2 = 21$, find the value of $2x + 3y$.

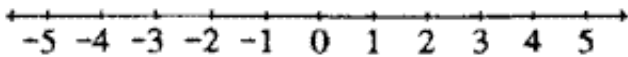
18. There are between 1961 and 2001 books in Luther's library. Exactly $1/7$ of them are textbooks, and exactly 20% of them are reference books. How many books are in Luther's library?

19. In a rectangle, the ratio of the width to the perimeter is $1 : 9$. Find the ratio of the width to the length. Express as a ratio $w : l$ in lowest terms.

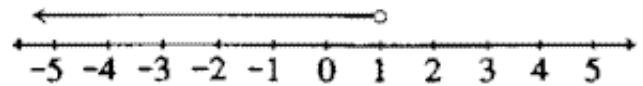
20. Shown on the right are two models of number graphs. Solve

$$\frac{x+3}{x+1} \leq 0$$

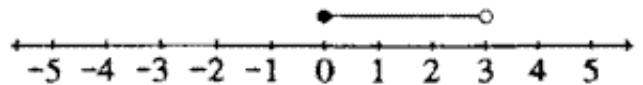
for all real numbers and graph using a number line like the one below. Please mark your answer ABOVE the number line.



The graph of $\{x : x < 1\}$:



The graph of $\{x : 0 \neq x < 3\}$:



21. Ms. Teller invested a total of \$25,000, part at 7% simple interest, and the rest at 9% simple interest. If her total income for one year from this interest was \$2001, how much did she invest at 9%?

22. $2\sqrt{2}$ is what percent of $8\sqrt{8}$?

23. Solve for x in terms of n and simplify: $(8^{4n})(32x) = 4^{8n}$. Use the form $x = 2^k$. Now write an expression for k .

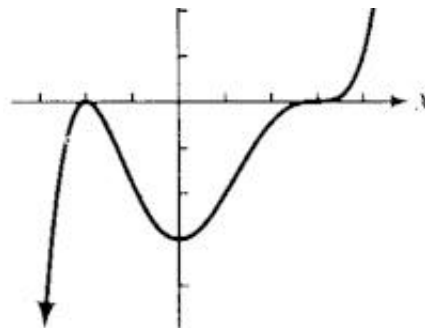
24. Positive integers a, b, c are said to form a Pythagorean triple if $a^2 + b^2 = c^2$. Write two distinct Pythagorean triples in the form a, b, c (with $a < b < c$) such that the perimeter ($a + b + c$) of a right triangle with side lengths a, b, c equals 60.

25. The sum of two numbers is 6 and their product is 10. Find the sum of their reciprocals.



26. In right triangle ABC , $m\angle A = 20.01^\circ$ and the right angle is at C . Find the numerical value of $\sin^2 A + \sin^2 B + \sin^2 C$.

27. See figure. Shown is the graph of the fifth degree polynomial $f(x) = k(x + 2)^a(x - 3)^b$. Find a and b .



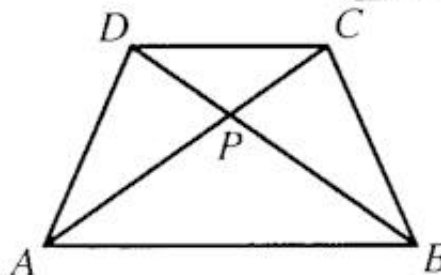
28. Find the volume of a right circular cylinder of height 3 cm and radius 4 cm. Be sure to include the correct units.

29. r varies directly as v and inversely as w . If $r = 4/3$ and $v = 9/14$ when $w = 3/7$, find w when $r = \sqrt{48}$ and $v = \sqrt{75}$.

#31

30. Find the temperature at which Fahrenheit and Celsius measurements have the same value.

31. See figure. Trapezoid $ABCD$ was formed by truncating an isosceles triangle with base \overline{AB} . The area of the original triangle was 72 cm^2 . The area of trapezoid $ABCD$ is 54 cm^2 . Determine the area (in cm^2) of triangle ABP .



32. Given the hyperbola $\frac{(x - 3)^2}{9} - \frac{(y + 2)^2}{16} = 1$, find the equation of the asymptote that passes closer to the origin. Express in the form $y = mx + b$.

#34

33. There are two positive numbers that may be inserted between 3 and 9 such that the first three numbers are in geometric progression, and the last three numbers are in arithmetic progression. Find those two numbers.

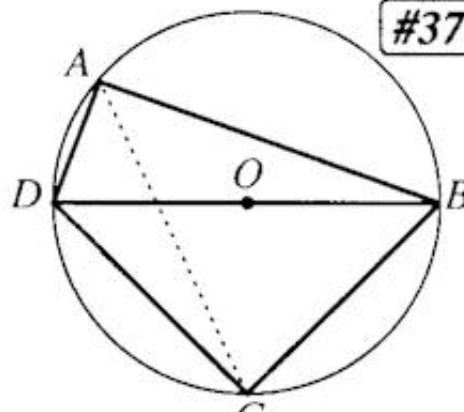
$$x = \frac{\begin{vmatrix} 5 & 1 & -1 \\ -7 & -2 & 3 \\ -8 & 1 & 2 \end{vmatrix}}{\begin{vmatrix} 3 & 1 & -1 \\ 1 & -2 & 3 \\ 2 & 1 & 2 \end{vmatrix}}$$

34. The class was asked to solve a system of three equations for (x, y, z) . Student A. Rith Mettick used Cramer's rule and solved for x as shown on the right. Now solve for z . Do not leave as a determinant.

35. Female mathematics teachers Susan Oravitz, Janice Osterloh, Beatrice Tehi, and Shirley Wallrabenstein, and male mathematics teachers Albert Laux, Dan Scott, and Bob Stover are serving on a statewide mathematics curriculum committee. Two of these women and two of these men are to be selected to attend a meeting in Columbus. How many different committees can be selected?

#37

36. Mathematics instructors (and site directors) Margaret Garner, Cathy Stoffer, Lanny Navarro, and Richard Morris together bought a cabin on the lake for \$120,000. Ms. Garner paid one half of the sum of the amounts paid by the other three; Ms. Stoffer paid one third of the sum of the amounts paid by the other three; and Mr. Navarro paid one fifth of the sum of the amounts paid by the other three. How much did Mr. Morris pay?



37. See figure. Given circle O as shown, $AD = 1$, $AB = 7$, and

$DC = BC$. Find AC .

38. A number system with base 60 was used by an early civilization. Many historians believe that 60 was chosen because it has so many integral factors. We still use this sexagesimal scale (base 60) when measuring time and degrees in minutes and seconds. We also owe to this civilization our present division of the circumference of a circle into 360 equal parts (360°). Name this civilization.

39. Solve for all real values of x : $(\log_3 x)(\log_x 5) = \log_3 5$.

40. Contest Coordinator Michael and former Contest Director Dick met with present Contest Director Duane in Bluffton. Michael left Cincinnati at 8:00 AM and drove 140 miles traveling at 60 mph (on I-75). Dick left Berea and drove 160 miles at 54 mph. Michael and Dick arrived in Bluffton at exactly the same time. To the nearest minute, at what time did Dick leave Berea? (Include AM or PM.)



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During the test, each student is permitted to have any handheld calculator(s), **including** the TI-89, TI-92 and HP95. Calculators with cordless transmission capabilities must be taped over.

On the answer sheet, please give your gender (for statistical purposes only.) Also we ask for your **home address** and **phone number**, so that you personally can be quickly notified and invited to attend the OHMIO competition, should you qualify for such. (Replaces [paragraph from 2000](#).)

Instructions:

- Place each answer on its proper blank on the answer sheet.
- Write the empty set as \emptyset or $\{ \}$. **NO CREDIT** given for $\{\emptyset\}$.
- Write multiple solutions as (e.g.) " $\{2, 3\}$ " **OR** " $x = 2$ or $x = 3$ " **OR** " $2, 3$ ".
NO CREDIT given for ordered pair form " $(2,3)$ ".
NO CREDIT given for " $x = 2$ **and** $x = 3$."
- In problems 1-20, **EXACT ANSWERS IN SIMPLEST FORM** are necessary.
For example:
 - write " $1 + \sqrt{2}$ " (not 2.414...);
 - write " $1/4$ " (not 0.785398...);

- write " $x = 5$ or $x = 1$ " (not 3 ± 2);
 - write '1' (not x^0);
 - write " $4/9$ " or " $0.\overline{4}$ " (not $16/36$, nor $(2/3)^2$, nor 0.4444).
- After problem 20, if an exact answer is not specified, then the answer may be written in **exact decimal, radical or fractional form**, or decimal form **rounded off to four places after the decimal point**.
 - The questions are not arranged according to difficulty. (There are some easy problems after number 30. Check it out!)
 - Testing time: **60 MINUTES**

Grading:

- Each correct answer counts one point. No partial credit will be given.
- There is no penalty for guessing.
- There may be one or more questions which are "impossible." In such an event, write "impossible" or "no solution." **NO CREDIT** given for \emptyset . (Replaces [paragraph from 2000](#).)



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