

# OCTM Math Contest 2000

THE OHIO COUNCIL OF TEACHERS OF MATHEMATICS  
 Twenty-Seventh Annual Contest  
 February 26, 2000

## Instructions

- Evaluate:  $20^{(0+0)}$
- Name the largest even prime number which is less than 2000.
- Find the total number of diagonals in a non-isosceles trapezoid.
- Give an example of a positive integer that is not a natural number.
- What is the first whole number which when spelled out contains the letter "a"? Write the answer as a numeral.
- Only one positive integer is exactly twice the sum of its digits. Find this two-digit number.
- Given:  $m \dots n$  and  $m \dots -n$ . Write the CAPITAL LETTER of the expression which has a different value than the other four expressions:

A)  $\frac{m-n}{m+n}$     B)  $\frac{m+n}{m-n}$     C)  $\frac{n-m}{-(m+n)}$     D)  $\frac{n-m}{-m-n}$     E)  $-\frac{n-m}{m+n}$

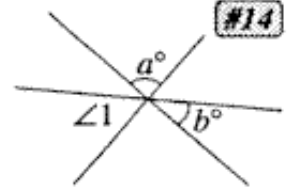
- Write the CAPITAL LETTER of the number which is incorrectly expressed:

- |    |       |                                       |
|----|-------|---------------------------------------|
| A) | 1234  | One thousand, two hundred thirty-four |
| B) | 0.018 | Eighteen thousandths                  |
| C) | 371   | Three hundred and seventy-one         |
| D) | 92    | Ninety-two                            |
| E) | 44.13 | Forty-four and thirteen hundredths    |

- The length of the base of an isosceles triangle is 2000 cm. Taking one exterior angle at each vertex, find the total number of degrees in the SUM of the measures of the exterior angles of that triangle.
- Ms. Math says, "I have \$2000. If I give you \$313, then you will have one-half as much money as I will. How much money do you have to begin with?"
- Johnny bought 40 benches at \$50 each. Parke bought 60 benches at \$40 each. What was the average cost per bench?
- Ms. Wytookay has twice as many nickels as she has dimes. The value of her nickels is \$20.00. Find the value of the dimes that Ms. Wytookay has.

13. It takes  $1\frac{1}{4}$  hours to go 60 miles. At the same rate, how many minutes will it take to go 80 miles?

14. See figure. Write the measure of angle 1 in terms of  $a$  and  $b$ .



15. The cost to participate in this contest is \$5.00 total for the first 4 students, and then \$2.00 for each additional student. Math teacher Connie Contest paid \$27.00 to register her team. How many students did she register?

16. Over the past four years of this contest, the winning scores have been:

1996: 37 by Ron Purewal (Toledo St. Francis DeSales)

1997: 36 by Andrew Schepler (Beavercreek) and Kai Sung (Thomas Worthington)

1998: 38 by James Telescan (Reynoldsburg)

1999: 39 by Allen Piscitello (Sycamore) and Kai Sung (Thomas Worthington)

Of these four scores let  $x$  = the median, and let  $y$  = the mean. Write the value of  $x - y$ .

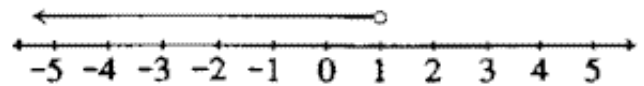
17. How many of the first one hundred positive integers are divisible by all of the numbers 2,3,4,5?

18. Find the area of the triangle whose vertices are at (0, 1), (1, 4), and (2, 3).

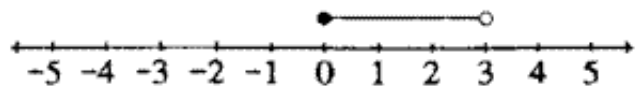
19. Three candidates ran for president of OCTM. Duane won the election with 64% of the votes. Bill had 750 votes, and Billie had 6% of the votes. Write the number of votes that president Duane received.

20. Shown on the right are two models of number graphs. Follow these models and, using a number line like the one below, show the graph of the intersection of the sets  $\{x : |x + 1| < 2\}$  and  $\{x : |x - 1| \leq 2\}$ .

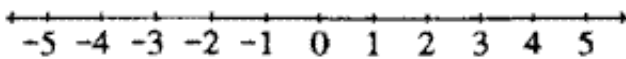
The graph of  $\{x : x < 1\}$ :



The graph of  $\{x : 0 \leq x < 3\}$ :



Please mark your answer ABOVE the number line.



21. On the day you were born, your generous grandparents invested \$19.00 in the stock market for you. Today that investment is worth \$38019.00. What is the total percent of increase on that investment?

22. Find the value of  $n$  such that  $n! = (3!)(5!)(7!)$ .

23.  $2^x 3^y 5^z = 2000$ , where  $x$ ,  $y$ , and  $z$  are integers. Find the value of  $2x + 3y + 5z$ .

24. If  $|x + y + 6| + |2x - y + 6| = 0$ , write the value of  $3xy$ .

25. Evaluate  $(x \dots 0)$ :  $1^x - 3x^0 + 3/9^{-1}$

26. Math teachers Michael Anderson, Jay Waggoner, Sandy Sikorski, Vilma Kolacz-Belanger, Ann Wildermuth, and Mark Jaffee were seated in a circle, evenly spaced, at the OCTM Annual Conference. Mr.

Anderson was opposite Mr. Jaffee, and Ms. Wildermuth was NOT opposite Ms. Kolacz-Belanger nor opposite Mr. Waggoner. Give the last names of all teachers who could NOT be seated next to Mr. Anderson.

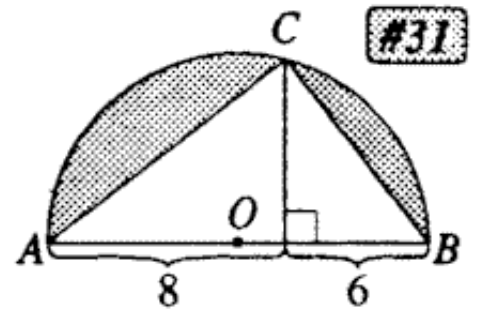
27. Find the least possible integral value of  $x$  such that  $\log_4 x + \log_3 81 > \log_2 32$ .

28. Find the total surface area of a right circular cylinder of height 4 cm and radius 3 cm. Be sure to include the correct units.

29. The graph of the cubic function  $f(x) = x^3 + ax^2 + bx + c$  crosses the  $x$ -axis at the points  $(-1, 0)$ ,  $(2, 0)$ , and  $(-3, 0)$ . Find the value of  $1 + a + b + c$ .

30. The symbol  $[x]$  represents the greatest integer which is less than or equal to  $x$ , e.g.,  $[4.2] = 4$ ,  $[7] = 7$ , and  $[-6.8] = -7$ . Find the value of  $x$  such that the product of  $x$  and  $[x]$  is 27.

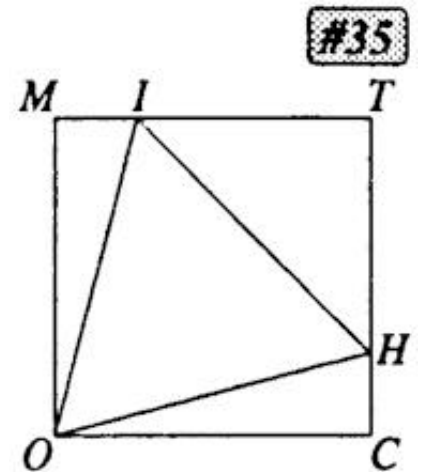
31. See figure.  $\triangle ABC$  is inscribed in semicircle  $O$ . Find the sum of the areas of the shaded regions.



32. If  $\sqrt[n]{2000} \# n \# \sqrt{2000}$ , write the number of possible integral values of  $n$ .

33.  $\sin A + \cos A = 1.38331$ , and  $\cos^2 A - \sin^2 A = 0.40674$ . Find  $A$  ( $0^\circ < A < 360^\circ$ ), correct to the nearest degree.

34. Solve for  $y$  in terms of  $x$  and  $z$ :  $z \sqrt{\frac{x}{y}} = \sqrt{z + \frac{x}{y}}$  (with  $y \dots 0$ ).



35. See figure. The area of square  $OCTM$  is 1. Find the area of equilateral triangle  $OHI$ .

36. Given the parabola  $y^2 - 8x - 6y - 3 = 0$ , find the distance between the focus and the vertex.

37. The winner of baseball's World Series is the team that first wins four games. Suppose that the Indians and the Reds meet in the World Series. If the probability that the Reds win each game is  $1/2$ , and the probability that the Indians win each game is  $1/2$ , what is the probability that the World Series will go to seven games before a champion is determined?

38. I was born in France in 1776, one year before the birth of the great German mathematician, Carl Friedrich Gauss. In the early 1800s I derived some general results from Fermat's proposed problems which were instrumental in helping many mathematicians throughout the years, and which helped Andrew Wiles in proving Fermat's Last Theorem in 1993. Early in life I became entranced by Gauss' masterpiece, *Disquisitiones Arithmeticae*. I wrote to Gauss, but fearing that he might be prejudiced against a woman mathematician, I assumed the name, 'Monsieur Leblanc.' When I was forced to drop my disguise and divulge my true name and identity to Gauss (whom I never met face-to-face), he wrote to me, paying me a supreme compliment: "When a person of the sex which, according to our customs and prejudices, must encounter infinitely more difficulties than men to familiarize herself with these thorny researches, succeeds nevertheless in surmounting these obstacles and penetrating the most obscure parts of them, then without doubt she must have the noblest courage, quite extraordinary talents, and a superior genius." Who am I?

39. 250 people are trapped in a ski lodge. There is enough food to last 8 days. If 50 experienced skiers are allowed to leave, how many days will that food last for the ones still in the lodge?

40. Math teacher Allen Burns left Toledo at 2 PM and traveled at 40 mph. Father Thomas O'Neill left from the same place at 3 PM, traveling the same route. He overtook Mr. Burns at 8 PM. What was Father O'Neill's average speed in miles per hour?



## OCTM Math Contest 2000 Instructions

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During the test, each student is permitted to have any handheld calculator(s), **including** the TI-89, TI-92 and HP95. Calculators with cordless transmission capabilities must be taped over.

On the answer sheet please give your current or most recent math course(s), the name and model number of your calculator(s), your race, and your gender. These items are for statistical purposes **ONLY**. (Changed for [2001](#).)

### Instructions:

- Place each answer on its proper blank on the answer sheet.
- Write the empty set as  $\emptyset$  or  $\{ \}$ . **NO CREDIT** given for  $\{\emptyset\}$ .
- Write multiple solutions as (e.g.) " $\{2, 3\}$ " **OR** " $x = 2$  or  $x = 3$ " **OR** " $2, 3$ ".  
**NO CREDIT** given for ordered pair form " $(2,3)$ ".  
**NO CREDIT** given for " $x = 2$  **and**  $x = 3$ ."
- In problems 1-20, **EXACT ANSWERS IN SIMPLEST FORM** are necessary.  
For example:
  - write " $1 + \sqrt{2}$ " (not 2.414...);
  - write " $1/4$ " (not 0.785398...);
  - write " $x = 5$  or  $x = 1$ " (not  $3 \pm 2$ );
  - write '1' (not  $x^0$ );
  - write " $4/9$ " or " $0.\overline{4}$ " (not  $16/36$ , nor  $(2/3)^2$ , nor 0.4444).

- After problem 20, if an exact answer is not specified, then the answer may be written in exact decimal, radical or fractional form, or decimal form rounded off to four places after the decimal point.
- The questions are not arranged according to difficulty.  
(There are some easy problems after number 30. Check it out!)
- Testing time: **60 MINUTES**

**Grading:**

- Each correct answer counts one point. No partial credit will be given.
- There is no penalty for guessing.
- There may be one or more questions which are "impossible." In such an event, write "impossible," "no answer," or "no solution." **NO CREDIT** given for  $\emptyset$ . (Changed for [2001](#).)



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This page revised on Sunday, March 02, 2003 09:40:12 -0500 .

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