2011 OHMIO PRESSURE ROUND QUESTIONS AND SOLUTIONS

1. $\sin(150^\circ) + 5 * \tan(135^\circ) + 5 \bullet \cos(300^\circ) = \csc(M)$, $0^\circ \le M < 360^\circ$. Find all values of M.

Answer: 210, 330 or 210°, 330°

Solution: $\frac{1}{2} + 5[-1] + 5\left[\frac{1}{2}\right] = \csc M \qquad -2 = \csc M \qquad \sin M = -\frac{1}{2} \qquad M = 210^\circ,\ 330^\circ$

2. Find the vertex of the parabola $2y^2 - 8y + 33 = -3x$. Express in ordered pair form (x,y).

Answer: $\left(\frac{-25}{3}, 2\right)$

Solution:

$2y^2 - 8y = -3x - 33 \qquad 2\left(y^2 - 4y + 4\right) = -3x - 25 \qquad 2\left(y\text{-}2\right)^2 = -3\left(x + \frac{25}{3}\right)$

$\{y - 2\}^2 = \frac{-3}{2}\left(x + \frac{25}{3}\right) \qquad \text{vertex} \left(\frac{-25}{3}, 2\right)$

3 $2011_6 = b_7$ Find b.
(The subscripts "6" and "7" mean that those numbers are written in base 6 and base 7).

Answer: b=1165

Solution

$2 \bullet 6^3 + 1 \bullet 6 + 1 \bullet 1 = 432 + 6 + 1 = 439 \qquad 7^2 = 49 \quad 7^3 = 343$

$439 = 1 \bullet 7^3 + 1 \bullet 7^2 = 6 \bullet 7 + 5 \qquad b = 1165$

4. In the rectangular coordinate plane, any circle which passes through (1.-2) and (3,1) cannot also pass through (2011,y). Find the value of y.

Answer: 3013

Solution: Three points determine a circle if they are not collinear. y cannot have the value that places it on a line through the given two points.

$y - 1 = \frac{1+2}{3-1}(x - 3) \qquad y = \frac{3}{2}x - \frac{7}{2} \qquad \text{y} = \frac{3}{2}(2011) - \frac{7}{2} = \frac{6033 - 7}{2} = 3013$

5. Find all positive integral values of n for which the expression $(n^3 - 12)/(n - 4)$

Has an integral value.

Answer: 2, 3, 5, 6, 8, 17,30, 56

Solution:

$$\frac{n^3-12}{n-4}=\frac{n^3-64}{n-4}+\frac{52}{n-4}=\frac{(n-4)(n^2+4n+16)}{(n-4)}+\frac{52}{(n-4)}=n^2+4n+16+\frac{52}{n-4}$$

$52 = 2 \bullet 2 \bullet 13$, The fraction has an integral value when $(n-4) = -2,\ -1,\ 1,\ 2,\ 4,\ 13,\ 26,$ or 52.

Therefore $n =$ 2, 3, 5, 6, 8, 17, 30, or 56

6. If $\log_2\left[\log_3\left(\log_3 b\right)\right]=\log_3\left[\log_2\left(\log_2 a\right)\right]=0,$ find the ratio of a to b.
Express in simplest form.

Answer: 4/27

Solution: $\log_3\left(\log_3 b\right)=1 \quad \therefore \log_3 b = 3$ and $b = 27 \quad \log_2\left(\log_2 a\right)=1 \quad \therefore \log_2 a = 2$ and $a = 4$
$\therefore\ b/a = 27/4$

7. Basketball player James Ayesmith has made 150 free throws in 200 attempts

for a free throw average of .750. In today's game, he will have 4 free throws.

What is the probability in lowest terms that he will make exactly 3 free throws?

Answer: 27/64

Solution: The four ways that he can make exactly 3 free throws are MHHH, HMHH, HHMH, and HHHM. H has probability of ¾ and M has a probability of ¼. The probability is $4*(3/4)^3(1/4)=27/64$

8. One ordered triple (x, y, z) which satisfies the system

$$\begin{aligned} x-2y+kz&=0\\ 2x-y+3z&=0\\ 3x-3y-2z&=0 \end{aligned}$$

is $(0,0,0)$.

Find the only value of k for with this system will have a solution different from (0, 0, 0)

Answer: -5

Solution: Eliminating x from (1) and (2) we get $3y+(3-2k)z=0,$ and
eliminating x from (1) and (3) we get $3y+(-3k-2)z=0,$ and solving these 2 together we get
$(3-2k+3k+2)=0,$ and $k=-5$

9. If $2x^3 - 5x^2 + 9$ is written in the form $p(x-3)^3 + q(x-3)^2 + r(x-3) + s,$ find the numerical value of the sum $p+q+r+s$.

Answer: 57

Solution: If $2x^3 - 5x^2 + 9$ is equal to $px^3 + qx^2 + rx + s$ moved to the right by 3 units, then $px^3 + qx^2 + rx + s$ is equal to $2x^3 - 5x^2 + 9$ moved to the left by 3 units or

$$2(x+3)^3 - 5(x+3)^2 + 9 = 2(x^3 + 9x^2 + 27x + 27) - 5(x^2 + 6x + 9) + 9 =$$

$$2x^3 + 13x^2 + 24x + 18 = px^3 + qx^2 + rx + s \quad \therefore\ p+q+r+s = 57$$

10. Express as a single radical in simplest form. $(\sqrt{2})(\sqrt[3]{4})(\sqrt[4]{8})$

Answer: $2\left(\sqrt[12]{2^{11}}\right) = 2\left(\sqrt[12]{2048}\right)$

Solution:.

$$2^{\frac{1}{2}} \bullet 2^{\frac{2}{3}} \bullet 2^{\frac{3}{4}} = 2^{\frac{23}{12}} = 2\left(\sqrt[12]{2^{11}}\right) = 2\sqrt[12]{2048}$$